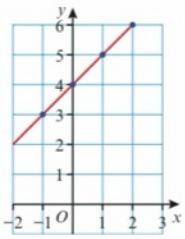
Chapter 10

Getting started

1



2 **a** y-value is 4 more than the x-value.

b $y = x + 4$

3 Gradient = 3

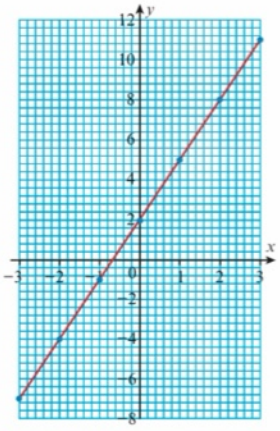
y-intercept = −2

Equation of the line is $y = 3x - 2$

Exercise 10.1

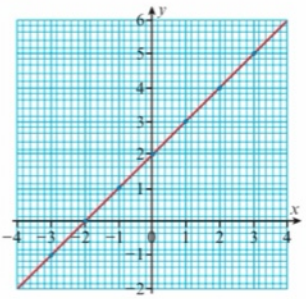
1 **a**

x	−3	−2	−1	0	1	2	3
y	−7	−4	−1	2	5	8	11



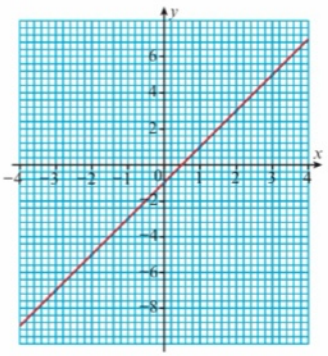
b

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



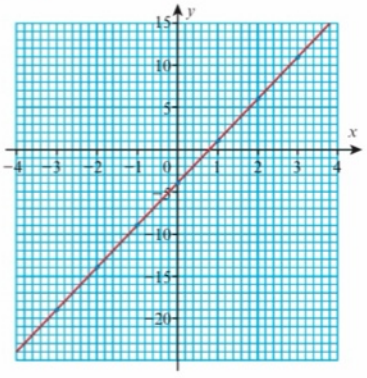
c

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5



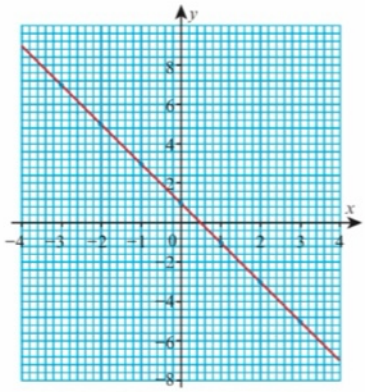
d

x	−3	−2	−1	0	1	2	3
y	−19	−14	−9	−4	1	6	11



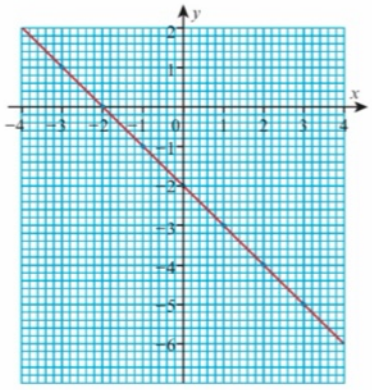
e

x	−3	−2	−1	0	1	2	3
y	7	5	3	1	−1	−3	−5



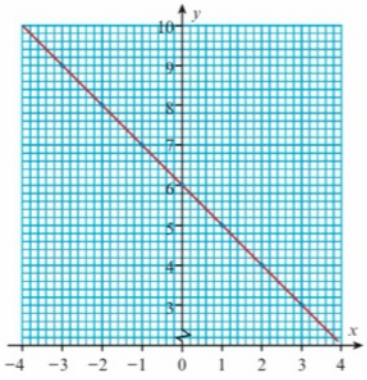
f

x	−3	−2	−1	0	1	2	3
y	1	0	−1	−2	−3	−4	−5



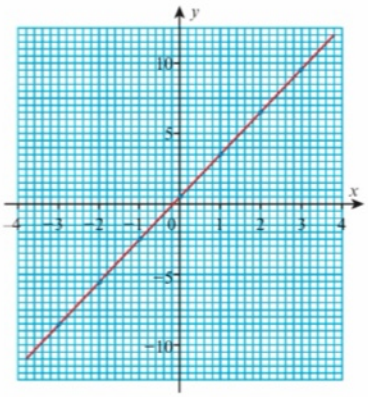
g

x	−3	−2	−1	0	1	2	3
y	9	8	7	6	5	4	3



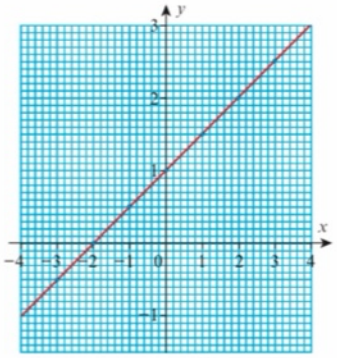
h

x	−3	−2	−1	0	1	2	3
y	−8.5	−5.5	−2.5	0.5	3.5	6.5	9.5



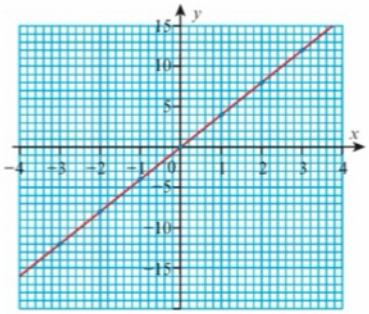
i

x	−3	−2	−1	0	1	2	3
y	−0.5	0	0.5	1	1.5	2	2.5



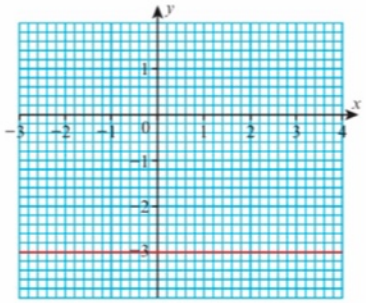
j

x	−3	−2	−1	0	1	2	3
y	−12	−8	−4	0	4	8	12



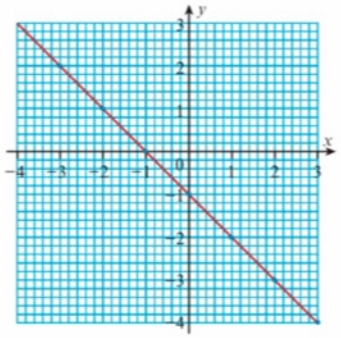
k

x	−3	−2	−1	0	1	2	3
y	−3	−3	−3	−3	−3	−3	−3



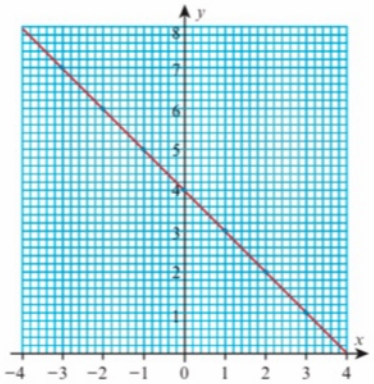
l

x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4



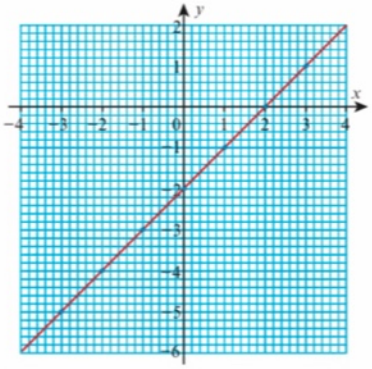
m

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1



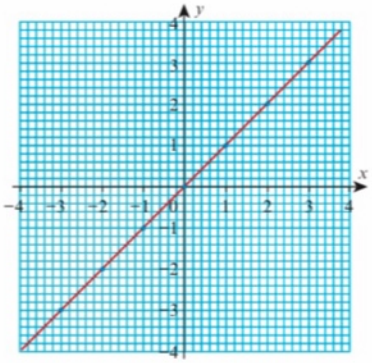
n

x	−3	−2	−1	0	1	2	3
y	−5	−4	−3	−2	−1	0	1



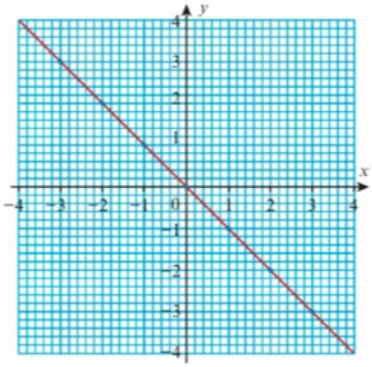
o

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

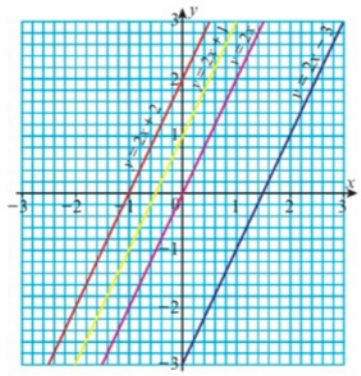


p

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3



2



The lines are parallel.

3 **a**

x	−3	0	3
$y = x + 2$	−1	2	5

b

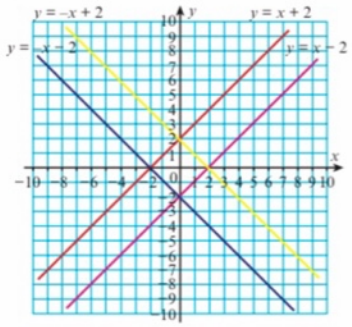
x	−3	0	3
$y = -x + 2$	5	2	−1

c

x	−3	0	3
$y = x - 2$	−5	−2	1

d

x	−3	0	3
$y = -x - 2$	1	−2	−5



4 **a** $y = x + 2$ cuts the x-axis at $x = -2$
$y = -x + 2$ cuts the x-axis at $x = 2$
$y = x - 2$ cuts the x-axis at $x = 2$
$y = -x - 2$ cuts the x-axis at $x = -2$

b $y = x + 2$ and $y = x - 2$

c $-x + 2$ and $-x - 2$

d $y = x + 2$ and $y = -x + 2$

e $y = x - 2$ and $y = -x - 2$

f None of the graphs

g $y = x + 2$ is parallel to $y = x - 2$
$y = -x + 2$ is parallel to $y = -x - 2$

h Same coefficients of x but different constant values.

Exercise 10.2

1 **a** $x = -4$

b $x = 2$

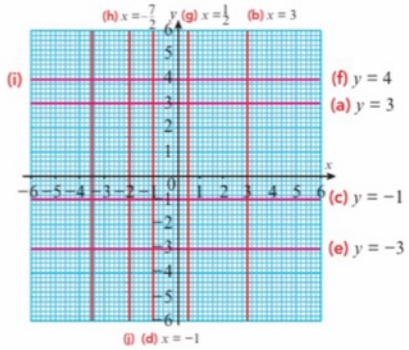
c $x = 7$

d $y = 7$

e $y = 3$

f $y = -6$

2



Exercise 10.3

1 **a** 3

b 2

c 1

d −3

e −5

f −1

g $\frac{1}{3}$

h $\frac{2}{3}$

i $-\frac{1}{4}$

2 **a** 3

b 1

c 2

d −3

e −3

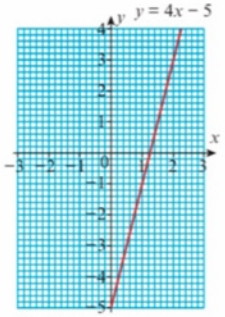
f $\frac{17}{4}$

3 450 m

The gradient is equal to the coefficient of x in the equation of the line.

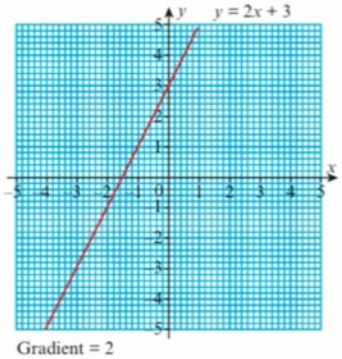
Exercise 10.4

1 a



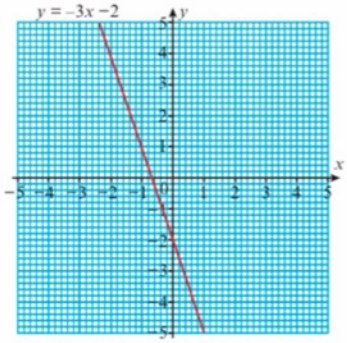
Gradient = 4
y-intercept = −5

b



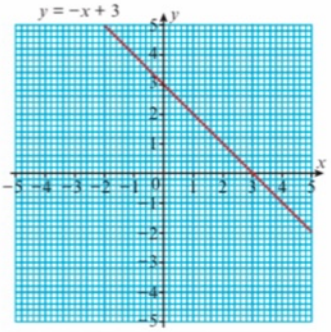
Gradient = 2
y-intercept = 3

c



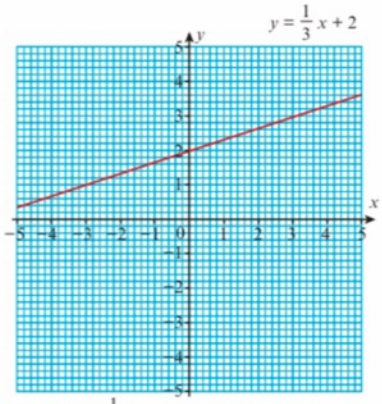
Gradient = −3
y-intercept = −2

d



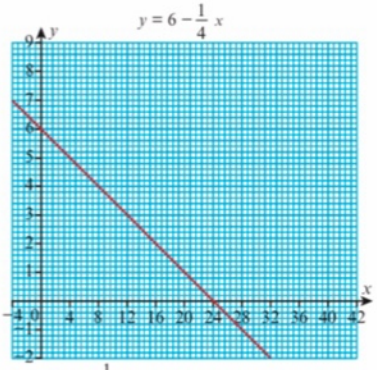
Gradient = −1
y-intercept = 3

e



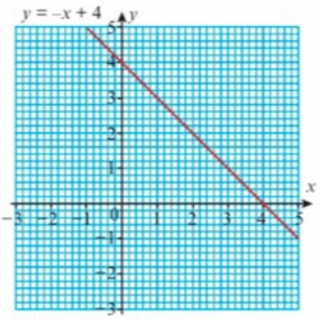
Gradient = $\frac{1}{3}$
y-intercept = 2

f



Gradient = $\frac{1}{4}$
y-intercept = 6

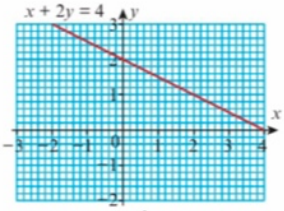
2 a



x-intercept $= -1$

y-intercept $= 4$

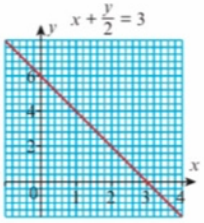
b



x-intercept $= -\frac{1}{2}$

y-intercept $= 2$

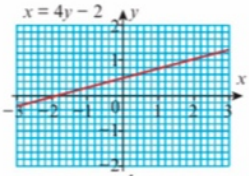
c



x-intercept $= -2$

y-intercept $= 6$

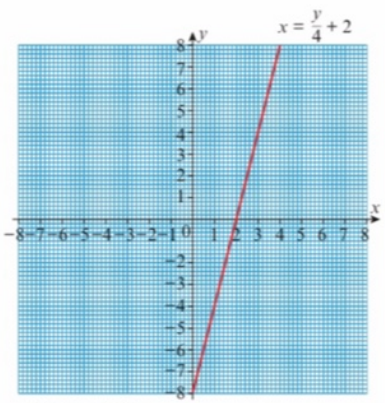
d



x-intercept $= \frac{1}{4}$

y-intercept $= \frac{1}{2}$

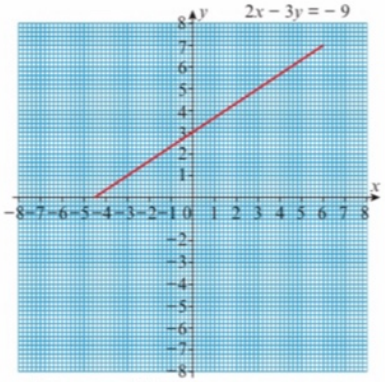
e



x-intercept $= 4$

y-intercept $= -8$

f



x-intercept $= -4.5$

y-intercept $= 3$

3

	$y = mx + c$	Gradient	y-intercept
a	$y = \frac{1}{2}x - 2$	$\frac{1}{2}$	-2
b	$y = -2x + 1$	-2	1
c	$y = 2x + 4$	2	4
d	$y = 2x - 5$	2	-5
e	$y = 2x + 5$	2	5
f	$y = -\frac{1}{3}x + 2$	$-\frac{1}{3}$	2
g	$y = 3x - 2$	3	-2
h	$y = -4x + 2$	-4	2
i	$y = 2x + 4$	2	4
j	$y = 6x - 12$	6	-12
k	$y = \frac{1}{8}x - 3$	$\frac{1}{8}$	-3
l	$y = -12x + 6$	-12	6

4 **a** $y = 2x + 3$
b $y = -3x - 2$
c $y = 3x - 1$
d $y = 0.75x - 0.75$
e $y = -2$
f $y = 4$

5 **a** $y = -\frac{3}{2}x - 0.5$
b $y = -\frac{3}{4}x + 2$
c $y = \frac{1}{2}x - 3$

6 **a** $y = -4x - 1$
b $y = \frac{1}{3}x + 1$
c $y = -3x + 2$
d $y = 5x + 2$
e $y = 3x + 1$
f $y = -x + 2$
g $y = 2x - 3$
h $y = \frac{2}{3}x - 1$
i $y = \frac{1}{4}x - 2$

7 **a** $y = 4x - 5$
b $y = -3x + 17$
c $y = \frac{9}{5}x - \frac{6}{5}$
d $y = \frac{17}{4}x - \frac{71}{4}$

8 Any line with the same gradient, e.g.
a $y = -3x - 5$
b $y = 2x + 13$
c $y = \frac{x}{2} - 3$
d $y = -x - 4$
e $x = -8$
f $y = 6$

9 **a**, **c**

10 **a** $y = 2x - 2$
b $y = 2x$
c $y = 2x - 4$
d $y = 2x + \frac{1}{2}$

11 **a** Any line with gradient $\frac{2}{3}$, e.g. $y = \frac{2}{3}x - 5$
b Any line with same y-intercept, e.g. $y = 2x + 3$
c $y = 3$

Investigation

No. Imagine a straight road with a straight footbridge crossing above it. Both the road and the bridge follow a straight line path, but they do not cross because there is vertical distance between them.

Exercise 10.5

1 $y = -5x + 8$

2 **a** Gradient $AB = -2$;
Gradient $PQ = \frac{1}{2}$; $-2 \times \frac{1}{2} = -1$,
so AB is perpendicular to PQ.
b Gradient $MN = \frac{1}{2}$; $\frac{1}{2} \times -2 = -1$,
so MN is perpendicular to AB.

3 $y = -\frac{1}{3}x + 5$

4 **a** $y = -\frac{1}{2}x + \frac{1}{2}$ or $x + 2y - 1 = 0$
b $y = x - 3$

5 Gradient $A = -2$, gradient $B = \frac{1}{2}$:
$-2 \times \frac{1}{2} = -1$, so A is perpendicular to B.

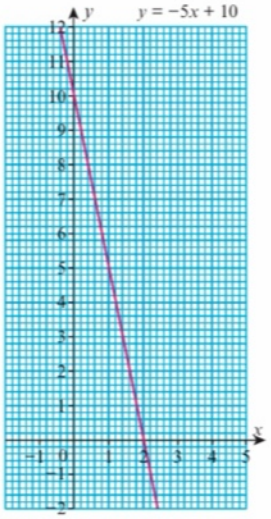
6 Gradient $AB = \frac{10}{9}$; gradient $AC = -1$, so AB is not perpendicular to AC and figure cannot be a rectangle.

7 **a** $y = 5x - 18$
b $y = x - 4$
c $y = 1$

8 $2x - y + 6 = 0$

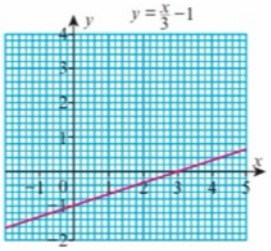
Exercise 10.6

1 a



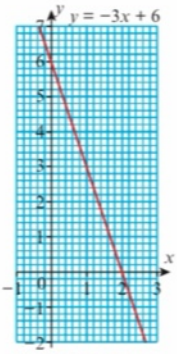
x-intercept = 2
y-intercept = 10

b



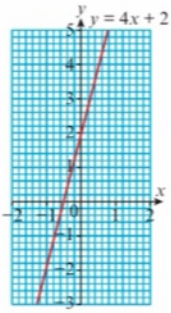
x-intercept = 3
y-intercept = −1

c



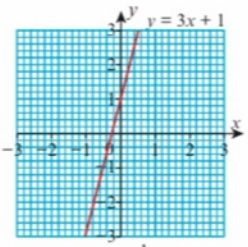
x-intercept = 2
y-intercept = 6

d



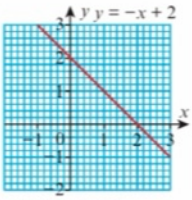
x-intercept = –0.5
y-intercept = 2

e



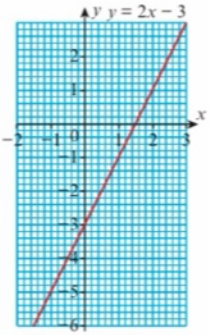
x-intercept = $-\frac{1}{3}$
y-intercept = 1

f



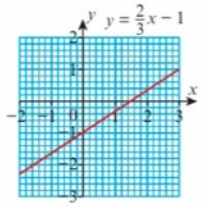
x-intercept = 2
y-intercept = 2

g



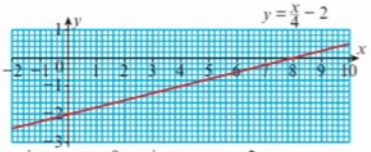
x-intercept = 1.5
y-intercept = –3

h



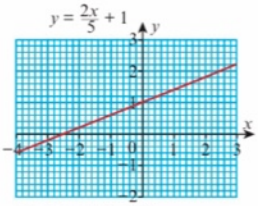
x-intercept = 1.5
y-intercept = −1

i



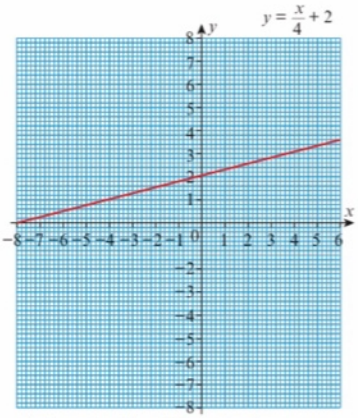
x-intercept = 8 y-intercept = −2

j



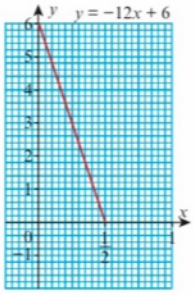
x-intercept = −2.5
y-intercept = 1

k



x-intercept = −8 y-intercept = 2

l



x-intercept = 0.5
y-intercept = 6

2 **a** $c = 2$
b $c = -4$
c $c = -9$
d $c = -8$
e $c = 4$
f $c = 3$
g $c = -2$
h $c = 2$

Exercise 10.7

1 **a** Length = 8.49 midpoint = (6, 9)
b Length = 4.47 midpoint = (3, 8)
c Length = 5.66 midpoint = (6, 5)
d Length = 3.16 midpoint = (4.5, 9.5)
e Length = 5 midpoint = (2.5, 5)
f Length = 1.41 midpoint = (11.5, 3.5)
g Length = 5 midpoint = (1, 3.5)
h Length = 6.08 midpoint = (4.5, 2)
i Length = 11.05 midpoint = (−2.5, 1.5)

2 $AB = 5.39$ midpoint = (3, 4.5)
$CD = 4.47$ midpoint = (−4, 6)
$EF = 8.60$ midpoint = (−2.5, 2.5)
$GH = 7.07$ midpoint = (3.5, 0.5)
$IJ = 5.10$ midpoint = (2.5, −3.5)
$KL = 12.6$ midpoint = (1, −3)
$MN = 5.39$ midpoint = (−3.5, −2)
$OP = 7.81$ midpoint = (−4.5, −4)

3 5.83

4 B

5 B

6 $AB = 6.40$
$AC = 4.24$
$BC = 7.28$

7 $a = 7$

8 $E = (-6, -2)$

Exercise 10.8

1 a $x^2 + 4x + 3$
b $x^2 + 10x + 24$
c $x^2 + 19x + 90$
d $x^2 + 15x + 36$
e $x^2 + 2x + 1$
f $x^2 + 9x + 20$
g $x^2 - 3x - 28$
h $x^2 + 5x - 24$
i $x^2 - 1$
j $x^2 - x - 72$
k $x^2 - 13x + 42$
l $x^2 - 9x - 52$
m $y^2 - 11y - 42$
n $z^2 - 64$
o $t^2 + 13t - 68$
p $h^2 - 6h + 9$
q $g^2 + 3\frac{1}{2}g - 2$
r $d^2 - \frac{9}{16}$

2 a $12 - 7x + x^2$
b $3 + 7x - 6x^2$
c $6m^2 - 17m + 7$
d $-8x^2 + 2x + 3$
e $8a^2 - 2b^2$
f $-8m^2 - 2mn + 3n^2$
g $x^2 + \frac{3}{4}x + \frac{1}{8}$
h $2x^2 - \frac{2}{3}x - \frac{1}{6}$
i $-36b^2 - 26b + 42$
j $6x^2 + 9x - 15$
k $6x^3 + 9x^2 + 2x + 3$
l $15x^4 - 18x^2 + 3$

3 a $2x^2 + 9x + 9$
b $3y^2 + 10y + 7$
c $7z^2 + 15z + 2$
d $4t^2 + 17t - 15$
e $2w^2 - 23w + 56$
f $16g^2 - 1$
g $72x^2 + 23x - 4$
h $360c^2 - 134c + 12$
i $-2m^2 + 10m - 12$

4 a $-2x^4 + 6x^2y - 4y^2$
b $-4x^4 + 2xy^2 - 4x^3y + 2y^3$
c $6x^3 + 9x^2y - 2xy - 3y^2$

5 a $15x^3 + 21x^2 - 24x - 12$
b $x^3 - 5x^2 - 25x + 125$
c $12x^3 + x^2 - 9x + 2$
d $4x^3 + 32x^2 + 80x + 64$
e $12x^3 - 32x^2 + 25x - 6$
f $18x^3 - 33x^2 + 20x - 4$
g $x^3 + 6x^2 + 12x + 8$
h $8x^3 - 24x^2 + 24x - 8$
i $x^4y^4 - x^4$
j $\frac{1}{81} - \frac{x^2}{18} + \frac{x^4}{16}$

6 a $V = \left(2x + \frac{1}{2}\right)(x - 2)^2\,\text{cm}^3$
b $2x^3 - 7.5x^2 + 6x + 2$
c $0.196\,\text{cm}^3$

Exercise 10.9

1 a $x^2 - 2xy + y^2$
b $a^2 + 2ab + b^2$
c $4x^2 + 12xy + 9y^2$
d $9x^2 - 12xy + 4y^2$
e $x^2 + 4xy + 4y^2$
f $y^2 - 8x^2y + 16x^4$
g $x^4 - 2x^2y^2 + y^4$
h $4 + 4y^3 + y^6$
i $4x^2 + 16xy^2 + 16y^4$
j $\frac{1}{4x^2} - \frac{1}{4xy} + \frac{1}{16y^2}$
k $\frac{9x^2}{16} - \frac{3xy}{4} + \frac{y^2}{4}$
l $a^2 + ab + \frac{b^4}{4}$
m $a^2b^2 + 2abc^4 + c^8$
n $9x^4y^2 - 6x^2y + 1$

5 B

6 $AB = 6.40$
$AC = 4.24$
$BC = 7.28$

7 $a = 7$

8 $E = (-6, -2)$

Exercise 10.8

1 a $x^2 + 4x + 3$
b $x^2 + 10x + 24$
c $x^2 + 19x + 90$
d $x^2 + 15x + 36$
e $x^2 + 2x + 1$
f $x^2 + 9x + 20$
g $x^2 - 3x - 28$
h $x^2 + 5x - 24$
i $x^2 - 1$
j $x^2 - x - 72$
k $x^2 - 13x + 42$
l $x^2 - 9x - 52$
m $y^2 - 11y - 42$
n $z^2 - 64$
o $t^2 + 13t - 68$
p $h^2 - 6h + 9$
q $g^2 + 3\frac{1}{2}g - 2$
r $d^2 - \frac{9}{16}$

2 a $12 - 7x + x^2$
b $3 + 7x - 6x^2$
c $6m^2 - 17m + 7$
d $-8x^2 + 2x + 3$
e $8a^2 - 2b^2$
f $-8m^2 - 2mn + 3n^2$
g $x^2 + \frac{3}{4}x + \frac{1}{8}$
h $2x^2 - \frac{2}{3}x - \frac{1}{6}$
i $-36b^2 - 26b + 42$
j $6x^2 + 9x - 15$
k $6x^3 + 9x^2 + 2x + 3$
l $15x^4 - 18x^2 + 3$

3 a $2x^2 + 9x + 9$
b $3y^2 + 10y + 7$
c $7z^2 + 15z + 2$
d $4t^2 + 17t - 15$
e $2w^2 - 23w + 56$
f $16g^2 - 1$
g $72x^2 + 23x - 4$
h $360c^2 - 134c + 12$
i $-2m^2 + 10m - 12$

4 a $-2x^4 + 6x^2y - 4y^2$
b $-4x^4 + 2xy^2 - 4x^3y + 2y^3$
c $6x^3 + 9x^2y - 2xy - 3y^2$

5 a $15x^3 + 21x^2 - 24x - 12$
b $x^3 - 5x^2 - 25x + 125$
c $12x^3 + x^2 - 9x + 2$
d $4x^3 + 32x^2 + 80x + 64$
e $12x^3 - 32x^2 + 25x - 6$
f $18x^3 - 33x^2 + 20x - 4$
g $x^3 + 6x^2 + 12x + 8$
h $8x^3 - 24x^2 + 24x - 8$
i $x^4y^4 - x^4$
j $\frac{1}{81} - \frac{x^2}{18} + \frac{x^4}{16}$

6 a $V = \left(2x + \frac{1}{2}\right)(x - 2)^2\,\text{cm}^3$
b $2x^3 - 7.5x^2 + 6x + 2$
c $0.196\,\text{cm}^3$

Exercise 10.9

1 a $x^2 - 2xy + y^2$
b $a^2 + 2ab + b^2$
c $4x^2 + 12xy + 9y^2$
d $9x^2 - 12xy + 4y^2$
e $x^2 + 4xy + 4y^2$
f $y^2 - 8x^2y + 16x^4$
g $x^4 - 2x^2y^2 + y^4$
h $4 + 4y^3 + y^6$
i $4x^2 + 16xy^2 + 16y^4$
j $\frac{1}{4x^2} - \frac{1}{4xy} + \frac{1}{16y^2}$
k $\frac{9x^2}{16} - \frac{3xy}{4} + \frac{y^2}{4}$
l $a^2 + ab + \frac{b^4}{4}$
m $a^2b^2 + 2abc^4 + c^8$
n $9x^4y^2 - 6x^2y + 1$

o $\frac{4x^2}{9} + \frac{16xy}{3} + 16y^2$

p $x^2 - 6x + 9$

2 a $4x - 12$

b $2x^2 + 2x - 19$

c $2y^2 + 8x^2$

d $\frac{x^2}{2} + \frac{8x}{3} - 2$

e $6x^2 + 13.8x + 3.6$

f $-16x^2 + 8xy + 2x - 2y^2$

g $-x^2 + 3x - 22$

h $4x^2 - 12xy - 19y^2$

i $-2x^3 - x^2 - 17x$

j $4x^2 - 13x - 1$

3 a -49

b 9

c 66

d 36

e 0

f 321

Discussion

The two numbers still multiply to give the constant term, but no longer add to give the coefficient of x because they will be multiplied by the coefficients of x in the brackets.

For example, $(2x + 1)(3x + 2) = 6x^2 + 4x + 3x + 2 = 6x^2 + 7x + 2$. The 1 and 2 multiply to give the constant term of 2, but they do not add to the give the coefficient of x (which is 7). This is because the 1 and 2 are multiplied by $2x$ and $3x$ respectively.

Exercise 10.10

1 a $(x + 12)(x + 2)$

b $(x + 2)(x + 1)$

c $(x + 4)(x + 3)$

d $(x + 7)(x + 5)$

e $(x + 9)(x + 3)$

f $(x + 6)(x + 1)$

g $(x + 6)(x + 5)$

h $(x + 8)(x + 2)$

i $(x + 10)(x + 1)$

j $(x + 7)(x + 1)$

k $(x + 20)(x + 4)$

l $(x + 7)(x + 6)$

2 a $(x - 6)(x - 2)$

b $(x - 4)(x - 5)$

c $(x - 4)(x - 3)$

d $(x - 4)(x - 2)$

e $(x - 8)(x - 4)$

f $(x - 7)(x - 7)$

g $(x - 10)(x + 2)$

h $(x - 9)(x + 2)$

i $(x - 8)(x + 4)$

j $(x + 3)(x - 2)$

k $(x + 11)(x - 3)$

l $(x + 12)(x - 2)$

3 a $(y + 17)(y - 10)$

b $(p - 6)(p + 14)$

c $(x - 12)(x - 12)$

d $(t + 18)(t - 2)$

e $(v + 15)(v + 5)$

f $3(x + 4)(x + 3)$

g $5(x + 1)(x - 2)$

h $3(x - 5)(x + 2)$

i $3(x - 1)^2$

j $2(x - 9)(x + 2)$

k $-2(x + 3)(x + 4)$

l $(x - 10)(x + 10)$

Exercise 10.11

1 a $(x + 6)(x - 6)$

b $(p + 9)(p - 9)$

c $(w + 4)(w - 4)$

d $(q + 3)(q - 3)$

e $(k + 20)(k - 20)$

f $(t + 11)(t - 11)$

g $(x + y)(x - y)$

h $(9h + 4g)(9h - 4g)$

i $4(2p + 3q)(2p - 3q)$

j $(12s + c)(12s - c)$

k $(8h + 7g)(8h - 7g)$

l $3(3x + 4y)(3x - 4y)$

m $2(10q + 7p)(10q - 7p)$

n $5(2d + 5e)(2d - 5e)$

o $(x^2 + y^2)(x^2 - y^2)$

p $x(y - x)(y + x)$

2 a $(x + 3)^2$

b $(x + 2^)^2$

c $(x - 7)^2$

d $(x - 9)^2$

e $(6 + x)^2$
f $(7 + x)^2$
g $(2 - x)^2$
h $(5 - x)^2$
i $(2x + 5)^2$

3 71

4 6

Exercise 10.12

1 a $x = 0$ or $x = 9$
b $x = 0$ or $x = -7$
c $x = 0$ or $x = 21$
d $x = 4$ or $x = 5$
e $x = -7$ or $x = -1$
f $x = -3$ or $x = 2$
g $x = -2$ or $x = -1$
h $x = -10$ or $x = -1$
i $x = 3$ or $x = 4$
j $x = 6$ or $x = 2$
k $x = 10$ or $x = -10$
l $t = -18$ or $t = 2$
m $y = -17$ or $y = 10$
n $p = -14$ or $p = 6$
o $w = 12$

2 a $x = -5$ or $x = 2$
b $x = -2$ or $x = 1$
c $x = 1$ or $x = -10$
d $x = 4$ or $x = -4$
e $x = -9$ or $x = 4$
f $x = -4$ or $x = 4$
g $x = 3$ or $x = 1$
h $x = 12$ or $x = 2$
i $x = 2$ or $x = 1$

Practice questions

1 a $15x^2 + 2x - 8$
b $x^2 + 20x + 36$
c $4x^2 - 9$
d $12y^4 - 5y^2 - 3$

2 $x^2 - \frac{1}{x^2}$

3 a $4x^2 - 2x + 25$
b $2x^2 + 8$

4 a $7 + 4\sqrt{3}$
b $4 + 2\sqrt{3}$
c $\frac{2 + \sqrt{3}}{2}$

5 a $y = 1.5x + 3$
b $y = 3$
c $y = -4x - 4$
d $y = -\frac{x}{10} - 3\frac{1}{2}$
e $x = 3$
f $y = \frac{2x}{3} - 3$
g $y = -x$

6 a $y = 4x + 4$
b $y = -3x + 13$
c $y = 0.5x + 0.9$

7 a $(m - n)(m + n)$
b $10\,000 - 9$
c $(100 - 3)(100 + 3) = 97 \times 103$

8 a $(x - 17)(x + 2)$
b $(4x - 7y)(4x + 7y)$

9 $x = 2$ or $x = -14$

10 $27x^3 + 54x^2 + 36x + 8$

11 a i $6x(2x - 1)$
ii $(y - 6)(y - 7)$
iii $(d + 14)(d - 14)$
b i $x = 0$ or $x = \frac{1}{2}$
ii $y = 6$ or $y = 7$
iii $d = 14$ or $d = -14$

12 20 cm or 380 cm

13 2 or 4

14 $x = 7$ or $x = -2$

15 **a** $(x - 21)(x - 29)$

b $x = 21$ or $x = 29$

c $AD = 100 - 2x$ m

d Area $= 100x - 2x^2$

e $100x - 2x^2 = 1218$

$x = 21$ or $x = 29$

If $x = 21$, width $= 21$ m and length $= 58$ m

If $x = 29$, width $= 29$ m and length $= 42$ m

16 **a** $b = 6$

b $(7, 4)$

c $y = -2x + 8$

d 5 units^2

Practice questions worked solutions

1 **a** $(3x - 2)(5x + 4)$
$= 15x^2 - 10x + 12x - 8$
$= 15x^2 + 2x - 8$

b $(x + 2)(x + 18)$
$= x^2 + 2x + 18x + 36$
$= x^2 + 20x + 36$

c $(2x + 3)(2x - 3)$
$= 4x^2 + 6x - 6x - 9$
$= 4x^2 - 9$

d $(4y^2 - 3)(3y^2 + 1)$
$= 12y^4 - 9y^2 + 4y^2 - 3$
$= 12y^4 - 5y^2 - 3$

2 $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = x^2 - 1 + 1 - \frac{1}{x^2}$
$= x^2 - \frac{1}{x^2}$

3 **a** $(2x - 5)^2 = (2x - 5)(2x - 5)$
$= 4x^2 - 10x - 10x + 25$
$= 4x^2 - 20x + 25$

b $(x + 2)^2 + (x - 2)^2$
$= x^2 + 4x + 4 + x^2 - 4x + 4$
$= 2x^2 + 8$

4 **a** $(2 + \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} + 2\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

b $(1 + \sqrt{3})(1 + \sqrt{3}) = 1 + \sqrt{3} + \sqrt{3} + 3$
$= 4 + 2\sqrt{3}$

c $\frac{(2 + \sqrt{3})^2}{(1 + \sqrt{3})^2} = \frac{7 + 4\sqrt{3}}{4 + 2\sqrt{3}}$

$= \frac{(7 + 4\sqrt{3})(4 - 2\sqrt{3})}{(4 + 2\sqrt{3})(4 - 2\sqrt{3})}$

$= \frac{28 - 14\sqrt{3} + 16\sqrt{3} - 8 \times 3}{16 - 4 \times 3}$

$= \frac{4 + 2\sqrt{3}}{4}$

$= \frac{2 + \sqrt{3}}{2}$

5 **a** $y = \frac{3}{2}x + 3$

b $y = 3$

c $y = -4x - 4$

d $y = -\frac{1}{10}x - \frac{7}{2}$

e $x = 3$

f $y = \frac{2}{3}x - 3$

g $y = -x$

6 **a** $y = 4x + c$
At $x = 3$, $y = 16$,
$16 = 12 + c$
$c = 4$
So, $y = 4x + 4$

b $y = -3x + c$
At $x = 7$, $y = -8$,
$-8 = -21 + c$
$c = 13$
So, $y = -3x + 13$

c $y = 0.5x + c$
At $x = 3$, $y = 2.4$,
$2.4 = 1.5 + c$
$c = 0.9$
So, $y = 0.5x + 0.9$

7 $AB^2 = (9 - 2)^2 + (6 - 10)^2$
$= 49 + 16$
$= 65$

$AC^2 = (-6 - 2)^2 + (-4 - 10)^2$
$= 64 + 196$
$= 260$

$BC^2 = (-6 - 9)^2 + (-4 - 6)^2$
$= 225 + 100$
$= 325$

$AB^2 + AC^2 = 65 + 260$
$= 325$
$= BC^2$

Therefore the lengths satisfy Pythagoras' theorem

$\Rightarrow$ the triangle is right angled.

8 **a** $m^2 - n^2 = (m + n)(m - n)$

b $9991 = 10\,000 - 9$
$= 100^2 - 3^2$

c $9991 = 100^2 - 3^2$
$= (100 + 3)(100 - 3)$
$= 103 \times 97$

9 **a** $(x - 17)(x + 2)$

b $(4x)^2 - (7y)^2 = (4x + 7y)(4x - 7y)$

10 $x^2 + 12x = 28$
$x^2 + 12x - 28 = 0$
$(x + 14)(x - 2) = 0$
$x = -14$ or $x = 2$

11 $(3x + 2)^3 = (3x + 2)(3x + 2)(3x + 2)$
$= (9x^2 + 6x + 6x + 4)(3x + 2)$
$= (9x^2 + 12x + 4)(3x + 2)$
$= 27x^3 + 54x^2 + 36x + 8$

12 **a** **i** $12x^2 - 6x = 6x(2x - 1)$

ii $y^2 - 13y + 42 = (y - 7)(y - 6)$

iii $d^2 - 196 = d^2 - 14^2$
$= (d + 14)(d - 14)$

b Using previous answers:

i $6x(2x - 1) = 0$
$x = 0$ or $2x - 1 = 0$
$x = 0$ or $x = \frac{1}{2}$

ii $(y - 7)(y - 6) = 0$
$y = 6$ or $y = 7$

iii $(d + 14)(d - 14) = 0$
$d = -14$ or $d = 14$

13 Square $\Rightarrow$ all sides are equal in length
So, $2x^2 + 3x - 9 = x^2 - 3x + 7$
$x^2 + 6x - 16 = 0$
$(x + 8)(x - 2) = 0$
$x = -8$ or $x = 2$
If $x = -8$, $x^2 - 3x + 7 = 64 + 24 + 7 = 95 \Rightarrow$ square perimeter $= 4 \times 95$
$= 380$ cm

If $x = 2$, $x^2 - 3x + 7 = 4 - 6 + 7 = 5 \Rightarrow$ square perimeter $= 4 \times 5$
$= 20$ cm

14 $AB^2 = (a - 3)^2 + (4 - 3)^2$
$= a^2 - 6a + 9 + 1$
$= a^2 - 6a + 10$

But $AB = \sqrt{2} \Rightarrow a^2 - 6a + 10 = (\sqrt{2})^2$
$= 2$

Therefore, $a^2 - 6a + 8 = 0$
$(a - 4)(a - 2) = 0$
$a = 4$ or $a = 2$

15 $5^{x(x+3)} = (5^2)^{x^2 - x - 7}$
Therefore, $x(x + 3) = 2(x^2 - x - 7)$
$x^2 + 3x = 2x^2 - 2x - 14$
So, $x^2 - 5x - 14 = 0$
$(x - 7)(x + 2) = 0$
$x = 7$ or $x = -2$

16 **a** $x^2 - 50x + 609 = (x - 29)(x - 21)$

b $2x^2 - 100x + 1218 = 0$
$x^2 - 50x + 609 = 0$
$(x - 29)(x - 21) = 0$
$x = 29$ or $x = 21$

c $2AD = 100 - 2x$
so $AD = 50 - x$

d Area $= x(50 - x)$

e $x(50 - x) = 609$
$50x - 50x^2 = 609$
$x^2 - 50x - 609 = 0$
Therefore, $x = 29$ or $x = 21$
So, the rectangle is $21 \times (50 - 21) = 21 \times 29$ (or $29 \times (50 - 21) = 29 \times 21$)

17 **a** $\frac{b - 2}{11 - 3} = \frac{1}{2}$
$2b - 4 = 8$
$2b = 12$
$b = 6$

b $\left(\frac{3 + 11}{2}, \frac{6 + 2}{2}\right)$, i.e. $(7, 4)$

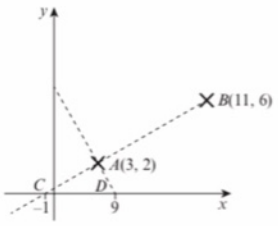
c Gradient of perpendicular line

$= -\frac{1}{\left(\frac{1}{2}\right)} = -2$

$y = -2x + c$ passes through (7, 4)

so, $4 = -14 + c \Rightarrow c = 18$

Therefore, $y = -2x + 18$



Line through A and B has equation

$y = \frac{1}{2}x + c$

and passes through (3, z),

so $2 = \frac{3}{2} + c \Rightarrow c = \frac{1}{2}$

Therefore, $y = \frac{1}{2}x + \frac{1}{2}$

This passes through x-axis. When $y = 0$,

$\frac{1}{2}x + \frac{1}{2} = 0 \Rightarrow x = -1$

Line through A perpendicular to AB passes through the x-axis when

$-2x + 18 = 0$

Therefore, $x = 9$.

$\text{Area} = \frac{1}{2}AD \times AC$

$= \frac{1}{2}\sqrt{(9-3)^2 + (2-0)^2}\sqrt{(3--1)^2 + (2-0)^2}$

$= \frac{1}{2}\sqrt{40}\sqrt{20}$

$= \frac{1}{2}\sqrt{8}\sqrt{5}\sqrt{4}\sqrt{5} = 10\sqrt{2}$