حل السؤال التاسع:

$$0 = 2\sin 2t$$

$$2t = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$3\cos t = 0$$

$$t = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{dy}{dx} = \frac{-3\sin t}{4\cos 2t}$$

$t = \frac{3\pi}{2}$ $\qquad$ $t = \frac{\pi}{2}$

$= \frac{3}{-4}$ $\qquad$ $= \frac{-3}{-4} = \frac{3}{4}$

حل ورقة عمل (4)

1) نشتق $\frac{dy}{dx} - 1 = -\sin y \frac{dy}{dx}$

نشتق مرة أخرى ←

$$\frac{d^2y}{dx^2} = -\sin y \frac{d^2y}{dx^2} - \cos y \left(\frac{dy}{dx}\right)^2$$

$$\cos y \left(\frac{dy}{dx}\right)^2 = -\sin y \frac{d^2y}{dx^2} - \frac{d^2y}{dx^2}$$

$$\cos y \left(\frac{dy}{dx}\right)^2 = -\frac{d^2y}{dx^2}(\sin y + 1)$$

بالقسمة على cos y :

$$\left(\frac{dy}{dx}\right)^2 = -\frac{d^2y}{dx^2}(\tan y + \sec y)$$

2) نأخذ ln الطرفين:

$$\ln y = \frac{1}{3}\left(\ln(1+x) - \ln(1-x^2)\right)$$

$$\frac{\frac{dy}{dx}}{y} = \frac{1}{3}\left(\frac{1}{1+x} - \frac{-2x}{1-x^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{3}y\left(\frac{1}{1+x} + \frac{2x}{1-x^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{3}\sqrt[3]{\frac{1+x}{1-x^2}} \times \frac{1-x+2x}{1-x^2}$$

$$= \frac{1}{3} \times \sqrt[3]{\frac{i+x}{1-x^2}} \times \frac{1+x}{1-x^2}$$

$$= \frac{1}{3}\left(\frac{1+x}{1-x^2}\right)^{\frac{4}{3}}$$

ص 1

3) $y^x = e^y - x - 1$

نأخذ ln الطرفين

$$x\ln y = \ln(e^y - x - 1)$$

نشتق: $\frac{xy'}{y} + \ln y = \frac{e^y y' - 1}{(e^y - x - 1)}$

$$\frac{1}{2\sqrt{a}} = \frac{1}{\sqrt{b}} \rightarrow \sqrt{b} = 2\sqrt{a} \quad \dots (1)$$

نعوض النقطة (a , b) بالمنحنى

$$\sqrt{a} + \sqrt{b} = 3$$

$$\sqrt{a} + 2\sqrt{a} = 3 \qquad \text{من (1) ...}$$

$$\sqrt{a} = 1 \rightarrow a = 1$$

$$\sqrt{b} = 2 \rightarrow b = 4$$

$$(1 , 4) \leftarrow$$

13) نجد المشتقة الأولى

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

نجد قيمة $\frac{dy}{dx}$ عند $(2 , 0)$:

$$4 + 0 - 2 - 4\frac{dy}{dx} = 0$$

$$2 = 4\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{1}{2}$$

نجد المشتقة مرة أخرى

$$2 + 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 - 4\frac{d^2y}{dx^2} = 0$$

نعوض: $x = 2 \ , \ y = 0 \ , \ \frac{dy}{dx} = \frac{1}{2}$

$$2 + \frac{1}{2} - 4\frac{d^2y}{dx^2} = 0$$

$$\frac{5}{2} = 4\frac{d^2y}{dx^2} \rightarrow \frac{d^2y}{dx^2} = \frac{5}{8}$$

14) نشتق $f'(y^2) \cdot 2y\frac{dy}{dx} = x\frac{dy}{dx} + y$

نعوض $x = 2 \ , \ y = 2$

$$f'(4) \cdot 4\frac{dy}{dx} = 2\frac{dy}{dx} + 2$$

$$12\frac{dy}{dx} - 2\frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{10} = \frac{1}{5}$$

15)

$$\frac{dx}{dt} = 3\cos 3t$$

$$\frac{dy}{dt} = -3\sin 3t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin 3t}{3\cos 3t} = -\tan 3t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-3\sec^2 3t}{3\cos 3t} = -\sec^3 3t$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = -\sec^3 \pi = 1$$

16)

$$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$$

لكن $\frac{dy}{dx} = 4$

$$2x + 8y - 8 = 0$$

$$x = 4 - 4y$$

نعوض في العلاقة $(4-4y)^2 + y^2 - 2y = 67$

$$16 - 32y + 16y^2 + y^2 - 2y = 67$$

$$17y^2 - 34y - 51 = 0$$

بالقسمة على 17 ← $y^2 - 2y - 3 = 0$

$$(y-3)(y+1) = 0$$

$$y = 3 \quad , \quad y = -1$$

$$(-8, 3) \quad , \quad (8, -1)$$

17) لتكن نقطة التماس (x, y)

ميل المماس عند تلك النقطة

$$\tan\frac{\pi}{4} = 1 = \frac{dy}{dx}$$

نجد الميل من المشتقة

$$2x + 2y\frac{dy}{dx} = 0 \rightarrow x + y\frac{dy}{dx} = 0$$

$$\therefore \quad \frac{dx}{dy} = \frac{-x}{y}$$

$$\frac{-x}{y} = 1 \rightarrow y = -x$$

نعوض في العلاقة

$$x^2 + x^2 = 8$$

$$2x^2 = 8 \rightarrow x^2 = 4$$

$$x = 2 \times, \quad -2 \checkmark \rightarrow (-2,2)$$

في الربع الثاني

معادلة المماس

$$y - 2 = 1(x + 2)$$

$$y = x + 4$$

يقطع محور x ← $y = 0$ $\quad$ $x = -4 \rightarrow (-4,0)$

يقطع محور y ← $x = 0$ $\quad$ $y = 4 \rightarrow (0,4)$

$$\text{Area} = \frac{1}{2}(0 - -4)(4 - 0)$$

$$= 8 \text{ وحدة مربعة}$$

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$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a(\cos\theta + \theta\sin\theta - \cos\theta)}{a(-\sin\theta + \theta\cos\theta + \sin\theta)}$$

$$\frac{dy}{dx} = \frac{\theta\sin\theta}{\theta\cos\theta} = \tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}} = \frac{\sec^2\theta}{a\theta\cos\theta}$$

$$a\theta\frac{d^2y}{dx^2} = \sec^3\theta \quad \#$$

7) نعلم أن : $(x-y)^{\text{زوجي}} = (y-x)^{\text{زوجي}}$

لذلك $2(x-y)^4 = 32$

$(x-y)^4 = 16$

نشتق الطرفين: $4(x-y)^3\left(1-\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx} = 1$ (a)

8) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dn}\left(\frac{dy}{dx}\right)}{\frac{dx}{dn}}$

$\frac{\frac{6n}{1}}{2n} = 12n^2\,|_{n=2} = 48$ (d)

9) نشتق: $1 = \cos 2y \,.\, 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2}\sec 2y$

$\frac{d^2y}{dx^2} = \frac{1}{2}\sec 2y \tan 2y \,.\, 2\frac{dy}{dx}$

$= \sec 2y \tan 2y \,.\, \frac{1}{2}\sec 2y$

$\frac{d^2y}{dx^2} = \frac{1}{2}\sec^2 2y \tan 2y$

بالقسمة على $\frac{1}{2}\sec^2 2y$

$2\cos^2 2y\frac{d^2y}{dx^2} = \tan 2y$

بالضرب بـ $\cos 2y$

$2\cos^3 2y\frac{d^2y}{dx^2} = \sin 2y = x$ (b)

10) $\ln x - \ln y = 2\ln y$

نشتق: $\ln x = 3\ln y$

$\frac{1}{x} = \frac{3}{y}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y}{3x}$ (b)

11) نشتق: $f'(y+1)\frac{dy}{dx} = 3x^2$

نعوض $y = 4$ لايجاد قيمة x

$f(5) = x^3 \rightarrow x^3 = 8 \rightarrow x = 2$

نعوض بالمشتقة

$f'(5)\frac{dy}{dx} = 12 \rightarrow \frac{dy}{dx} = \frac{12}{4} = 3$ (b)

12) نفرض النقطة $(a\,,\,b)$

نشتق $\frac{1}{2\sqrt{x}} + \frac{\frac{dy}{dx}}{2\sqrt{y}} = 0$

نعوض النقطة في المشتقة

$\frac{1}{2\sqrt{a}} - \frac{2}{2\sqrt{b}} = 0$

تبادلي

$$\frac{xy'}{y}(e^y - x - 1) + \ln y\,(e^y - x - 1) = e^y y' - 1$$

$$\ln y\,(e^y - x - 1) + 1 = e^y y' - \frac{x}{y}(e^y - x - 1)y'$$

عامل مشترك

$$\ln y\,(e^y - x - 1) + 1 = y'\left(e^y - \frac{x}{y}(e^y - x - 1)\right)$$

$$y' = \frac{\ln y(e^y - x - 1) + 1}{e^y - \frac{x}{y}(e^y - x - 1)}$$

(4

$$\cos(x+y)^2.2(x+y).\left(1 + \frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

$$2\cos(x+y)^2.(x+y) - 1 = \frac{dy}{dx}(1 - \cos(x+y)^2.2(x+y))$$

$$\frac{dy}{dx} = \frac{2\cos(x+y)^2.(x+y) - 1}{1 - 2\cos(x+y)^2.(x+y)} = -1$$

5) نشتق:

$$xe^y \frac{dy}{dx} + e^y = e^y \frac{dy}{dx}$$

$$\frac{dy}{dx}(xe^y - e^y) = -e^y$$

بالقسمة على e^y

$$\frac{dy}{dx}(x-1) = -1$$

$$\frac{dy}{dx} = \frac{-1}{x-1}$$

$$\frac{d^2y}{dx^2} = \frac{1}{(x-1)^2}$$

لكن $xe^y - e^y = 1$

$$\rightarrow\ e^y(x-1) = 1 \rightarrow x - 1 = \frac{1}{e^y}$$

$$\frac{d^2y}{dx^2} = \frac{1}{\left(\frac{1}{e^y}\right)^2} = e^{2y}$$

6) نشتق: $e^{xy}\left(x\frac{dy}{dx} + y\right) = 1 - \frac{dy}{dx}$

$$xe^{xy}\frac{dy}{dx} + \frac{dy}{dx} = 1 - e^{xy}y$$

$$\frac{dy}{dx}(xe^{xy} + 1) = 1 - e^{xy}y$$

$$\frac{dy}{dx} = \frac{1 - e^{xy}y}{xe^{xy} + 1}$$

نعوض $e^{xy} = x - y$

$$\frac{dy}{dx} = \frac{1 - (x-y)y}{x(x-y) + 1}$$

$$\frac{dy}{dx} = \frac{1 - xy + y^2}{x^2 - xy + 1}$$ #