ورقة عمل الدرس الأول

السـؤال الأول :

اختـر الاجابة الصحيحة لكل مما يلي :

1. $(\sin x + \cos x)^2 =$

a) $1 + \sin x \cos x$ b) $1 + \cos 2x$

c) $1 + 2\sin x \cos x$ d) $1 + \frac{1}{2}\sin 2x$

2. $\frac{1 - sinx}{1 + sinx} =$

a) $secx - \tan x$ b) $secx + \tan x$

c) $(secx - \tan x)^2$ d) $(secx + \tan x)^2$

3. $cot(-x)\cos(-x) + \sin(-x) =$

a) $secx$ b) $cscx$ c) $-\csc x$ d) $-secx$

4. $\sin^4 x - \cos^4 x =$

a) $2\sin x \cos x$ b) $-2\sin x \cos x$

c) $\cos^2 x - \sin^2 x$ d) $\sin^2 x - \cos^2 x$

5. $\frac{\tan 40° - \tan 10°}{1 + \tan 40° \tan 10°} =$

a) $\sqrt{3}$ b) $\frac{-1}{\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $-\sqrt{3}$

6. $ln|\tan x| =$

a) $ln|\sin x| + ln|\cos x|$ b) $ln|\sin x| + ln|secx|$

c) $ln|\cos x| - ln|\sin x|$ d) $ln|\sin x| + ln|cscx|$

7. $\frac{(\sin x + \cos x)^2 - 1}{2\sin x \cos x}$

a) 1 b) -1 c) 2 d) $\frac{1}{2}$

8. $\frac{1}{1 - \sin\theta} - \frac{1}{1 + \sin\theta} =$

a) $secxtan\theta$ b) $\frac{1}{2}secxtan\theta$

c) $2secxtan\theta$ d) $-2secxtan\theta$

السـؤال الثاني:

إذا علمت أن $\cos\theta = \frac{5}{13}$ حيث θ بالربع الرابع جد:

$$\frac{2-3\cot\theta}{4+9\sqrt{\sec^2\theta-1}}$$

السـؤال الثالث :

أثبت أن $\cos^2(A-120°)+\cos^2(A+120°)+\cos^2 A=\frac{3}{2}$

السـؤال الرابع :

إذا كان $\tan x-\tan y=a$ $\quad coty-cotx=b$

أثبت أن : $\frac{1}{a}+\frac{1}{b}=\cot(x-y)$

السـؤال الخامس:

إذا كان A, B, C زوايا مثلث أثبت أن :

$$\tan\frac{A}{2}=\cot\left(\frac{B+C}{2}\right)$$

السـؤال السادس :

أثبت صحة كل من المتطابقات الآتية :

1. $\sin(A+B)+\sin(A-B)=2\sin A\cos B$
2. $\sin A+\cos A=\sqrt{2}\sin\left(A+\frac{\pi}{4}\right)$
3. $g(x)=\sin x$ إذا كان

فأثبت أن :

$$\frac{g(x+x)-g(x)}{h}=\sin x\left(\frac{\cos h-1}{h}\right)+\cos x\left(\frac{\sin h}{h}\right)$$

9. $\tan\alpha=x+1$ إذا كان

$\tan\beta=x-1$ وكان

فإن $\cot(\alpha-\beta)$ يساوي :

a) $2x^2$ b) $\frac{1}{2}x^2$

c) x^2 d) $\frac{x^2-1}{2}$

10. $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)+\cos^{-1}\frac{\sqrt{3}}{2}\right)=$ إن

حيث الزوايا في الربع ألأول :

a) $\frac{1}{2}$ b) 1 c) $\frac{3}{4}$ d) $\frac{1}{4}$

11. إذا علمت أن $y=\tan\theta, x=\sin\theta$

فإن : $\frac{1}{x^2}-\frac{1}{y^2}=$

a) 1 b) 2 c) -1 d) -2

12. $(\sin x+\cos x)^2+(\sin x-\cos x)^2=$

a) 1 b) 2

c) $2\sin 2x$ d) $-2\sin 2x$

13. $\tan^2 A\sec^2 B-\sec^2 A\tan^2 B$

a) $\tan^2 A+\tan^2 B$ b) $\tan^2 A-\tan^2 B$

c) $\tan^2 A\tan^2\beta$ d) $2\tan^2 A-2\tan^2\beta$

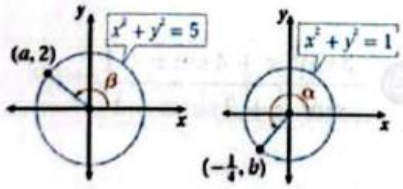
14. $\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\frac{\sin^3 A-\cos^3 A}{\sin A-\cos A}=$

a) 2 b) -2 c) 1 d) -1

السـؤال السابع :

مستخدماً الشكل المجاور لإيجاد كل من الاقترانات الآتية علماً بأن : $f(x)=\sin x, g(x)=\cos x$

$h(x)=\tan x$



1. $f(\alpha+\beta)$
2. $g(\alpha-\beta)$
3. $h(\alpha+\beta)$

السـؤال الثامن :

دون استخدام الآلة الحاسبة جد قيمة كل من النسب المثلثية الآتية :

1. $\tan 195^\circ$
2. $\tan 315^\circ$
3. $cot 135^\circ$
4. $\sin 165^\circ$
5. $\tan\left(\frac{19\pi}{12}\right)$
6. $sec\left(\frac{-\pi}{12}\right)$
7. $\sin\frac{\pi}{18}\cos\frac{5\pi}{18}+\cos\frac{\pi}{18}\sin\frac{5\pi}{18}$

السـؤال التاسع :

بسط كلاً من العبارات المثلثية :

1. $\frac{1}{1-\cos x}+\frac{1}{1+\cos x}$
2. $\frac{3\sin^2 x+4\sin x+1}{\sin^2 x+2\sin x+1}$
3. $\frac{\cos^2 x-1}{\cos^2 x-\cos x}$
4. $\frac{1+\sin x}{1-\sin x}$

اثباتات قدرات عليا (تميز)

1. إذا كان $\sin\theta=\frac{2pq}{p^2+q^2}$ بالربع الأول θ

أثبت أن $sec\theta+\tan\theta=\frac{p+q}{p-q}$

2. أثبت أن $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2csc\theta$

3. إذا علمت أن θ بالربع الأول أثبت أن :

$\tan\theta+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=sec\theta$

4 إذا علمت أن $\cos\theta + sec\theta = \sqrt{3}$ أثبت أن :

$$\cos^3\theta + sec^3\theta = 0$$

5 أثبت أن

$$\tan x + \tan 2x - \tan 3x = -\tan x \tan 2x \tan 3x$$

6 إذا كان A, B, C زوايا مثلث فأثبت أن :

$$\frac{\sin(B+C)+\sin(C+A)+\sin(A+B)}{\sin(\pi+A)+\sin(3\pi+B)+\sin(5\pi+C)} = -1$$

7 إذا كان $\cos x + \sec x = 2$ فأثبت أن :

$$\cos^8 x + \sec^8 x = 2$$

حل ورقة العمل (1)

السؤال الأول :

1 $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$ (منطابقة 1)

$= 1 + 2\sin x\cos x$ C

2 $\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$ تضرب بالمرافق

$= \frac{(1-\sin x)^2}{1-\sin^2 x} = \frac{1-2\sin x+\sin^2 x}{\cos^2 x}$ نوزع

$= \frac{1}{\cos^2 x} - 2\frac{\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$= sec^2 x - 2sec x\tan x + \tan^2 x$

$= (sec x - \tan x)^2$ C

3 $\cot(-x)\cos(-x) + \sin(-x)$

$= -\cot x.\cos x - \sin x$

$= -\frac{\cos x}{\sin x}.\cos x - \sin x = \frac{-\cos^2 x - \sin^2 x}{\sin x}$

$= \frac{-1}{\sin x} = -\csc x$ C

4 $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$ (1)

d

5 $\frac{\tan 40^\circ - \tan 10^\circ}{1+\tan 40^\circ \tan 10^\circ} = \tan(40^\circ - 10^\circ)$

$= \tan 30^\circ = \frac{1}{\sqrt{3}}$ c

6 $\ln\left|\frac{\sin x}{\cos x}\right| = \ln|\sin x| - \ln|\cos x|$

$\ln|\sin x| + \ln|\cos x|^{-1} = \ln|\sin x| + \ln|\sec x|$ b

7 $\frac{(\sin x+\cos x)^2 - 1}{\sin 2x} = \frac{\sin^2 x + 2\sin x\cos x + \cos^2 x - 1}{\sin 2x}$ (منطابقة 1)

$= \frac{1 + 2\sin x\cos x - 1}{2\sin x\cos x} = 1$ a

8 $\frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta} = \frac{1+\sin\theta-1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)} = \frac{2\sin\theta}{1-\sin^2\theta}$

$= \frac{2\sin\theta}{\cos^2\theta} = 2sec\theta\tan\theta$ c

9 $cot(\alpha-\beta) = \frac{1}{\tan(\alpha-\beta)} = \frac{1+\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$

$\frac{1+(x+1)(x-1)}{x+1-(x-1)} = \frac{1+x^2-1}{2} = \frac{x^2}{2}$ b

10 $\cos\left(\sin^{-1}\frac{1}{2}\right)\cos\left(\cos^{-1}\frac{5}{2}\right) - \sin\left(\sin^{-1}\frac{1}{2}\right)\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

$= \cos(30^\circ).\frac{\sqrt{3}}{2} - \sin(30^\circ)\sin(30^\circ)$

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ a

11 $\frac{1}{x^2} - \frac{1}{y^2} = \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta}$

$= csc^2\theta - cot^2\theta = 1$ a

12 $(\sin x+\cos x)^2 + (\sin x - \cos x)^2 =$

$\sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= 1 + 2\sin x\cos x + 1 - 2\sin x\cos x = 2$ b

السؤال الرابع :

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{\tan x-\tan y}+\frac{1}{\cot x-\cot y}$$

$$=\frac{1}{\frac{\cos y}{\sin y}-\frac{\cos x}{\sin x}}+\frac{1}{\frac{\cos x}{\sin x}-\frac{\cos y}{\sin y}}$$

$$=\frac{\cos x\cos y}{\sin x\cos y-\cos x\sin y}+\frac{\sin x\sin y}{\sin x\cos y-\cos x\sin y}$$

$$=\frac{\cos x\cos y+\sin x\sin y}{\sin x\cos y-\cos x\sin y}=\frac{\cos(x-y)}{\sin(x-y)}=\cot(x-y)$$

السؤال الخامس :

$$A+B+C=\pi\rightarrow A=\pi-(B+C)$$

$$\frac{A}{2}=\frac{\pi}{2}-\left(\frac{B+C}{2}\right)\leftarrow \text{نقسم على 2}$$

$$\tan\frac{A}{2}=\tan\left(\frac{\pi}{2}-\left(\frac{B+C}{2}\right)\right)\leftarrow \text{نأخذ tan}$$

$$\tan\frac{A}{2}=\cot\left(\frac{B+C}{2}\right)$$

السؤال السادس :

1 $\sin(A+B)+\sin(A-B)=$

$$\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B=2\sin A\cos B$$

2 نبدأ باليمين :

$$\sqrt{2}\sin\left(A+\frac{\pi}{4}\right)=\sqrt{2}[\sin A\cos\frac{\pi}{4}+\cos A\sin\frac{\pi}{4}]$$

$$=\sqrt{2}[\frac{1}{\sqrt{2}}\sin A+\frac{1}{\sqrt{2}}\cos A]=\sin A+\cos A$$

3 $$\frac{g(x+h)-g(x)}{h}=\frac{\sin(x+h)-\sin x}{h}$$

مشترك

$$=\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$=\frac{\sin x[\cos h-1]}{h}+\frac{\cos x(\sin h)}{h}$$

13 نستخدم العوامل المشتركة

$$\tan^2 A[1+\tan^2\beta]-[1+\tan^2 A]\tan^2\beta$$

$$=\tan^2 A+\tan^2 A\tan^2\beta-\tan^2\beta-\tan^2 A\tan^2\beta$$

$$=\tan^2 A-\tan^2\beta$$ **b**

14 نستخدم التحليل

$$\frac{(\sin A+\cos A)(\sin^2 A-\sin A\cos A+\cos^2 A)}{\sin A+\cos A}$$

$$+\frac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{\sin A-\cos A}$$

$$=\sin^2 A-\sin A\cos A+\cos^2 A+\sin^2 A+\sin A\cos A+\cos^2 A$$

$$=2\sin^2 A+2\cos^2 A=2$$ **a**

السؤال الثاني :

$$\sin^2\theta=1-\cos^2\theta=1-\frac{25}{169}=\frac{144}{169}\rightarrow \sin\theta=\frac{\pm 12}{13}$$

$$\sin\theta=-\frac{12}{13}\leftarrow \text{بالربع الرابع } \theta \text{ لكن}$$

$$\text{المطلوب}=\frac{2-3\times\frac{\frac{5}{13}}{-\frac{12}{13}}}{4+9\sqrt{\frac{169}{25}-1}}=\frac{2-3\times-\frac{5}{12}}{4+9\times\frac{12}{5}}=\frac{65}{512}$$

السؤال الثالث :

$$\cos(A-120^\circ)=\cos A\cos 120^\circ+\sin A\sin 120^\circ$$

$$=-\frac{1}{2}\cos A+\frac{\sqrt{3}}{2}\sin A$$

$$\cos(A+120^\circ)=\cos A\cos 120^\circ-\sin A\sin 120^\circ$$

$$=-\frac{1}{2}\cos A-\frac{\sqrt{3}}{2}\sin A$$

$$\cos^2(A-120^\circ)+\cos^2(A+120^\circ)+\cos^2 A$$

$$=\left(-\frac{1}{2}\cos A+\frac{\sqrt{3}}{2}\sin A\right)^2+\left(-\frac{1}{2}\cos A-\frac{\sqrt{3}}{2}\sin A\right)^2+\cos^2 A$$

$$=\frac{1}{4}\cos^2 A-\frac{\sqrt{3}}{2}\sin A\cos A+\frac{3}{4}\sin^2 A$$

$$+\frac{1}{4}\cos^2 A+\frac{\sqrt{3}}{2}\cos A\sin A+\frac{3}{4}\sin^2 A+\cos^2 A$$

$$=\frac{3}{2}\cos^2 A+\frac{3}{2}\sin^2 A=\frac{3}{2}$$

6 $\sec\left(\frac{-\pi}{12}\right) = \sec\left(\frac{\pi}{12}\right) = \frac{1}{\cos(15°)}$

$= \frac{1}{\cos(45° - 30°)} = \frac{1}{\cos 45° \cos 30° + \sin 45° \sin 30°}$

$= \frac{1}{\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}} = \frac{2\sqrt{2}}{\sqrt{3}+1}$

7 $\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18}$

$\sin 10 \cos 50 + \cos 10 \sin 50 = \sin(60) = \frac{\sqrt{3}}{2}$

السؤال التاسع :

1 $\frac{1+\cos x + 1 - \cos x}{(1-\cos x)(1+\cos x)} = \frac{2}{1-\cos^2 x}$

$= \frac{2}{\sin^2 x} = 2csc^2 x$

2 $\frac{(3\sin x + 1)(\sin x + 1)}{(\sin x + 1)^2} \cdot \frac{\sin x - 1}{\sin x - 1}$

$= \frac{(3\sin x + 1)(\sin x - 1)}{-\cos^2 x} = -\frac{3\sin^2 x - 2\sin x - 1}{\cos^2 x}$

$= -3\tan^2 x + 2sec x \tan x + sec^2 x$

3 $\frac{\cos^2 x - 1}{\cos^2 x - \cos x} = \frac{(\cos x - 1)(\cos x + 1)}{\cos x[\cos x - 1]}$

$1 + sec^2 x$ نوزع البسط على المقام

4 $\frac{1+\sin x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{1 + 2\sin x + \sin^2 x}{\cos^2 x}$

$sec^2 x + 2sec x \tan x + \tan^2 x =$ نوزع

السؤال السابع :

$a^2 + 4 = 5 \rightarrow a^2 = 1 \Leftarrow$ نجد a بالتعويض في المعادلة

$\therefore a = 1, -1$ لأنها في الربع الثاني $(-1, 2)$

× ✓

$\left(-\frac{1}{4}\right)^2 + b^2 = 1 \rightarrow b^2 = \frac{15}{16} \Leftarrow b$ نجد

$\therefore b = -\frac{\sqrt{15}}{4}$ لأنها تقع في الربع الثالث

$\sin\alpha = -\frac{\sqrt{15}}{4}, \cos\alpha = -\frac{1}{4}, \tan\alpha = \sqrt{15}$

$\sin\beta = \frac{2}{5}, \cos\beta = -\frac{1}{5}, \tan\beta = -2$

1 $f(\alpha+\beta) = \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{\sqrt{15}-2}{20}$

2 $g(\alpha-\beta) = \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin x \sin\beta = \frac{1-2\sqrt{15}}{20}$

3 $h(\alpha+\beta) = \tan(\alpha+\beta) = \frac{\sqrt{15}-2}{1+2\sqrt{15}}$

السؤال الثامن :

1 $\tan 195 = \tan(45° + 150°)$

$= \frac{\tan 45° + \tan 150°}{1 - \tan 45° \tan 150°} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

بضرب البسط والمقام ب $\sqrt{3}$

2 $\tan 315° = \tan(360° - 45°) = \frac{\tan 360° - \tan 45°}{1 + \tan 360° \tan 45°}$

$= \frac{-1}{1} = -1$

3 $cot 135° = -cot(180° - 45°) = -cot 45 = -1$

4 $\sin(165°) = \sin(120° + 45°) = \sin 120° \cos 45° + \cos 120° \sin 45°$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + -\frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

5 $\tan\left(\frac{19\pi}{12}\right) = \tan(285)$

$= \tan(45° + 240°) = \frac{\tan 45° + \tan 240°}{1 - \tan 45° \tan 240°}$

$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$

اثباتات قدرات عليا (تميز)

الحـل

1 $\cos^2\theta = 1 - \sin^2\theta$

$$\cos^2\theta = 1 - \frac{4p^2q^2}{(p^2+q^2)^2}$$

$$\cos^2\theta = \frac{(p^2+q^2)^2 - 4p^2q^2}{(p^2+q^2)^2}$$

$$\cos^2\theta = \frac{p^4+q^4+2p^2q^2-4p^2q^2}{(p^2+q^2)^2}$$

$$\cos^2\theta = \frac{(p^2-q^2)^2}{(p^2+q^2)^2} \rightarrow \cos\theta = \frac{p^2-q^2}{p^2+q^2}$$

$$\sec\theta + \tan\theta = \frac{p^2+q^2}{p^2-q^2} + \frac{\frac{2pq}{p^2+q^2}}{\frac{p^2-q^2}{p^2+q^2}}$$

$$= \frac{p^2+2pq+q^2}{p^2-q^2}$$

$$= \frac{(p+q)^2}{(p-q)(p+q)} = \frac{p+q}{p-q}$$

2 $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$

$$\frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta} = \frac{\overbrace{\sin^2\theta+\cos^2\theta}^{1} + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2[1+\cos\theta]}{(1+\cos\theta)\sin\theta} = \frac{2}{\sin\theta} = 2\csc\theta$$

3 $\tan\theta + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta}}$

$$= \tan\theta + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \tan\theta + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \tan\theta + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = sec\theta$$

---

4 $\cos^3\theta + sec^3\theta = 0$ المطلوب إثبات أن :

أي :

$$(\cos\theta + sec\theta)(\cos^2\theta - sec\theta\cos\theta + sec^2\theta)$$

$$(\cos\theta + sec\theta)(\cos^2\theta + sec^2\theta - 1) \ldots . \text{X}$$

لكن

$\cos\theta + sec\theta = \sqrt{3}$ (تربيع الطرفين)

$\cos^2\theta + 2sec\theta\cos\theta + sec^2\theta = 3$

$\boxed{\cos^2\theta + sec^2\theta = 1}$

نعوض في X

$\sqrt{3}[1-1] = 0$

---

5 $\tan 3x = \tan(2x+x) = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$\tan 3x = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ تبادلي

$\tan 2x + \tan x = \tan 3x - \tan 3x \tan 2x \tan x$

$\tan 2x + \tan x - \tan 3x = -\tan x \tan 2x \tan 3x$

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6 $\dfrac{\sin(B+C) + \sin(C+A) + \sin(A+B)}{\sin(\pi+A) + \sin(3\pi+B) + \sin(5\pi+C)}$

$= \dfrac{\sin(\pi-A) + \sin(\pi-B) + \sin(\pi-C)}{-\sin A - \sin B - \sin C}$ لأن

$A + B + C = \pi$

$= \dfrac{-\sin A - \sin B - \sin C}{-\sin A - \sin B - \sin C} = -1$

---

7 $\cos x + \sec x = 2$

$\cos x + \dfrac{1}{\cos x} = 2 \rightarrow \cos^2 x + 1 = 2\cos x$ (نضرب cosx)

$\cos^2 x - 2\cos x + 1 = 0 \rightarrow (\cos x - 1)^2 = 0$

$\boxed{\cos x = 1}$

$\cos^8 x + \sec^8 x = (1)^8 + \dfrac{1}{(1)^8} = 2$ نطبق

ورقة عمل (2)

1 إذا علمت أن $\cot\theta = \frac{2}{3}, \sin\theta > 0$ فجد قيمة :

1. $\sin\theta$
2. $\cos\theta$
3. $\sin 2\theta$
4. $\cos 2\theta$
5. $\sin\frac{\theta}{2}$
6. $\cos\frac{\theta}{2}$
7. $\tan 2\theta$
8. $sec2\theta$

2 دون استخدام الآلة الحاسبة جد ما يلي :

1. $\sin 15^\circ$
2. $\tan\frac{7\pi}{8}$
3. $\tan 202.5^\circ$
4. $\cos^2 37.5^\circ - \sin^2 37.5^\circ$
5. $\cos\left(\frac{23\pi}{12}\right)$

3 أثبت صحة كل من المتطابقات التالية :

1. $\cos^4 2x - \sin^4 2x = 1 - 2\sin^2 2x$
2. $\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$
3. $\frac{cot\theta - \tan\theta}{cot\theta + \tan\theta} = \cos 2\theta$
4. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$
5. $\frac{\sin 10x}{\sin 9x + \sin x} = \frac{\cos 5x}{\cos 4x}$

6. $\tan^2\left(\frac{x}{2} + \frac{\pi}{4}\right) = \frac{1+\sin x}{1-\sin x}$

7. $\alpha \varepsilon R - \{0, 2, -2\}$ إذا علمت أن

   $\tan x = \alpha + 1, \tan y = \alpha - 1$ وكان

   $\tan 2x - \tan 2y = \frac{4}{\alpha^2 - 4}$ أثبت أن

8. $\frac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x} = \tan 4x$ أثبت أن

9. $\tan x = \frac{1}{t + \sqrt{1+t^2}}, \tan y = \frac{1}{t - \sqrt{1+t^2}}$ إذا علمت أن

   $\tan(x+y) = -t$ أثبت أن

10. $\tan\frac{x}{2} = \pm\frac{1-\cos x}{\sin x}$ أثبت أن

11. $\frac{\sin 2x}{1+\cos 2x} \times \frac{\cos x}{1+\cos x} = \tan\left(\frac{x}{2}\right)$

4 اختر الاجابة الصحيحة فيما يلي (1 - 12) :

1. $\sin 15^\circ \cos 15^\circ$

   a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{2\sqrt{2}}$ d) $\frac{1}{8}$

2. $\sin^2 22.5^\circ \times \cos^2 22.5^\circ =$

   a) $\frac{1}{16}$ b) $\frac{1}{4}$ c) $\frac{1}{2\sqrt{2}}$ d) $\frac{1}{8}$

3. $180^\circ < x < 270^\circ$ إذا علمت أن $\tan x = \frac{4}{3}$

   فإن $\cos\frac{x}{2} =$

   a) $\frac{1}{\sqrt{5}}$ b) $-\frac{1}{\sqrt{5}}$ c) $-\sqrt{\frac{2}{5}}$ d) $-\sqrt{\frac{1}{10}}$

10 $\sin\frac{5\pi}{12}+\sin\frac{\pi}{12}=$

a) $-\sqrt{\frac{3}{2}}$ b) $\sqrt{\frac{3}{2}}$ c) $\sqrt{\frac{2}{3}}$ d) $-\sqrt{\frac{2}{3}}$

11 $\cos^4 x-\sin^4 x=$

a) $\cos 2x$ b) $-\cos 2x$ c) $\sin 2x$ d) $-\sin 2x$

12 يمكن كتابته على الصورة $sec2x$

a) $\frac{sec^2x csc^2x}{sec^2x+csc^2x}$ b) $\frac{-sec^2x csc^2x}{sec^2x-csc^2x}$

c) $\frac{sec^2x csc^2x}{secx-\tan x}$ d) $\frac{sec^2x csc^2x}{secx+\tan x}$

4 $\frac{\sin 4x+\sin 2x}{\cos 4x+\cos 2x}=$

a) $\tan 2x$ b) $cot2x$ c) $\tan 4x$ d) $\tan 3x$

5 تساوي $\frac{1-\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}$ ان

a) $\sin x$ b) $\cos x$ c) $secx$ d) $cscx$

6 تساوي $\frac{\sin x+\sin 3x}{\sin 2x}$ ان

a) $2\cos x$ b) $\cos x$ c) $2\sin x$ d) $1+\sin x$

7 تساوي $\sin 15^\circ \sin 45^\circ$ ان

a) $\frac{1}{8}$ b) $\frac{1-\sqrt{3}}{4}$ c) $\frac{\sqrt{3}-1}{4}$ d) $\frac{\sqrt{3}-1}{2}$

8 تساوي $\sin^2 15^\circ \times \sin^2 45^\circ$ ان

a) $\frac{1}{2}+\frac{\sqrt{3}}{8}$ b) $\frac{1}{4}-\frac{\sqrt{3}}{8}$ c) $\frac{1}{4}+\frac{\sqrt{3}}{8}$ d) $\frac{1}{2}-\frac{\sqrt{3}}{8}$

9 تساوي $\frac{\sin 3x}{\sin 2x}$ ان

a) $\cos x-secx$

b) $2\cos x-\frac{1}{2}secx$

c) $4\cos x-secx$

d) $3\cos x-\frac{1}{2}secx$

حل ورقة عمل (2)

السـؤال الأول :

1 $\cot\theta = \frac{2}{3}$

في الربع الأول $\sin\theta > 0$, $\cot\theta > 0$

$\csc^2\theta - \cot^2\theta = 1$

$\csc\theta = \frac{\sqrt{13}}{3}$ $\sin\theta > 0$

$\sin\theta = \frac{3}{\sqrt{13}}$

1 $\sin\theta = \frac{3}{\sqrt{13}}$

2 $\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\frac{9}{13}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$

لأن θ بالربع الأول

حيث $\cos > 0, \sin > 0$

3 $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2.\frac{3}{\sqrt{13}}.\frac{2}{\sqrt{13}} = \frac{12}{13}$

4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \frac{4}{13} - \frac{9}{13} = \frac{-5}{13}$

5 $\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\frac{2}{\sqrt{13}}}{2}} \approx 0.47$

6 $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1+\frac{2}{\sqrt{13}}}{2}} \approx 0.88$

7 $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\times\frac{3}{2}}{1-\frac{9}{4}} = \frac{3}{\frac{-5}{4}} = \frac{-12}{5}$

8 $sec2\theta = \frac{1}{\cos 2\theta}$

$= \frac{-13}{5}$

السـؤال الثاني :

1 $\sin(15°) = \sin\left(\frac{30°}{2}\right)$

$= \sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

2 $\tan\left(\frac{7\pi}{8}\right)$

$\frac{7\pi}{8} = 157.5 = \frac{315°}{2}$

$= \tan\left(\frac{\frac{7\pi}{4}}{2}\right) = -\sqrt{\frac{1-\cos\frac{7\pi}{4}}{1+\cos\frac{7\pi}{4}}}$

$\cos\left(\frac{7\pi}{4}\right) = \cos(315°)$ لكـن

$\cos(45°) = \frac{1}{\sqrt{2}}$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

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5 $\cos\left(\frac{23\pi}{12}\right)$

$$\frac{23\pi}{12} = 345$$

$$\cos(345) = \cos(15) = \cos\left(\frac{30^\circ}{2}\right) = \sqrt{\frac{1+\cos 30^\circ}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

السـؤال الثالث:

1 نحلل

$$\left(\cos^2 2x - \sin^2 2x\right)\left(\cos^2 2x + \sin^2 2x\right)$$

$$= \left(\left(1-\sin^2 2x\right) - \sin^2 2x\right)(1) = 1 - 2\sin^2 2x$$

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2 نبدأ بجهة اليمين

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1-\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}{1+\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}$$

$\cos^2\frac{\theta}{2}$ نضرب البسط والمقام ب

$$= \frac{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}$$

$$= \frac{\cos 2\left(\frac{\theta}{2}\right)}{1} = \cos\theta$$

$$\therefore \tan\left(\frac{7\pi}{8}\right) = -\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$$

$$= -\sqrt{\frac{\frac{\sqrt{2}-12}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}}$$

$$= -\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

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3 $\tan(202.5^\circ)$

$$\tan(202.5^\circ) = \tan\left(\frac{405^\circ}{2}\right)$$

(202.5°) في الربع الثالث

حيث $(405^\circ - 360^\circ = 45^\circ)$

$$= \sqrt{\frac{1-\cos 405^\circ}{1+\cos 405^\circ}}$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

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4 $\cos 2\theta$

$$\cos(2\times 37.5^\circ) = \cos(75)$$

$$= \cos\left(\frac{150}{2}\right)$$

(75°) في الربع الأول

$$= \sqrt{\frac{1+\cos 150}{2}}$$

$$\cos 150^\circ = -\cos 30^\circ$$

$$= \frac{2\alpha+2}{1-\alpha^2-2\alpha-1} - \frac{2\alpha-2}{1-\alpha^2+2\alpha-1}$$

$$\frac{2(\alpha+1)}{-\alpha(\alpha+2)} - \frac{2(\alpha-1)}{-\alpha(\alpha-2)} = \frac{2(\alpha-1)}{\alpha(\alpha-2)} - \frac{2(\alpha+1)}{\alpha(\alpha+2)}$$

$$= \frac{2(\alpha-1)(\alpha+2) - 2(\alpha+1)(\alpha-2)}{\alpha(\alpha^2-4)}$$

$$= \frac{2[\alpha^2+\alpha-2-\alpha^2+\alpha+2]}{\alpha(\alpha^2-4)} = \frac{2\times 2\alpha}{\alpha(\alpha^2-4)} = \frac{4}{\alpha^2-4}$$

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8 نبدأ من اليسار

$$\frac{(\sin 7x+\sin x)+(\sin 5x+\sin 3x)}{(\cos 7x+\cos x)+(\cos 5x+\cos 3x)}$$

$$= \frac{2\sin 4x\cos 3x + 2\sin 4x\cos x}{2\cos 4x\cos 3x + 2\cos 4x\cos x}$$

$$= \frac{2\sin 4x(\cos 3x+\cos x)}{2\cos 4x(\cos 3x+\cos x)}$$

$$= \tan 4x$$

هذا السؤال من الدرس الاول

9

$$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$$

$$= \frac{\frac{1}{t+\sqrt{1+t^2}} + \frac{1}{t-\sqrt{1+t^2}}}{1-\frac{1}{t+\sqrt{1+t^2}}\cdot\frac{1}{t-\sqrt{1+t^2}}}$$

نضرب البسط والمقام بالقوسين

$$\tan(x+y) = \frac{t-\sqrt{1+t^2}+\left(t+\sqrt{1+t^2}\right)}{\left(t+\sqrt{1+t^2}\right)\left(t-\sqrt{1+t^2}\right)-1}$$

$$= \frac{2t}{t^2-t\sqrt{1+t^2}+t\sqrt{1+t^2}-1-t^2-1}$$

$$= \frac{2t}{-2} = -t$$

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3 نبدأ بالجهة اليسرى

$$\frac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta} = \frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}} = \frac{\cos 2\theta}{1} = \cos 2\theta$$

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4 نوحد المقام

$$\frac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{\cos^2 x-\sin^2 x}$$

$$= \frac{1+\sin 2x-(1-\sin 2x)}{\cos 2x} = \frac{2\sin 2x}{\cos 2x} = 2\tan 2x$$

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5 $$\frac{\sin 10x}{\sin 9x+\sin x} = \frac{2\sin 5x\cos 5x}{2\cos 5x\sin 4x}$$

$$= \frac{\cos 5x}{\sin 4x}$$

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6 $$\tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right) = \tan^2\frac{1}{2}\left(x+\frac{\pi}{2}\right)$$

$$= \frac{1-\cos\left(x+\frac{\pi}{2}\right)}{1+\cos\left(x+\frac{\pi}{2}\right)}$$

$$= \frac{1-\left[\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}\right]}{1+\left[\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}\right]} = \frac{1+\sin x}{1-\sin x}$$

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7 $$\tan 2x-\tan 2y = \frac{2\tan x}{1-\tan^2 x} - \frac{2\tan y}{1-\tan^2 y}$$

$$= \frac{2(\alpha+1)}{1-(\alpha+1)^2} - \frac{2(\alpha-1)}{1-(\alpha-1)^2}$$

3 x في الربع الثالث

$$sec^2x = 1 + \tan^2 x = \frac{25}{9}$$

$$secx = -\frac{5}{3} \rightarrow \cos x = \frac{-3}{5}$$

$$\cos\frac{x}{2} = -\sqrt{\frac{1+-\frac{3}{5}}{2}} = -\sqrt{\frac{\frac{2}{5}}{2}} = \frac{-1}{\sqrt{5}}$$ (b)

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4 $$\frac{2\sin 3x\cos x}{2\cos 3x\cos x}$$

$$= \tan 3x$$ (d)

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5 $$\frac{1-\frac{\sin^2\frac{1}{2}x}{\cos^2\frac{1}{2}x}}{1+\frac{\sin^2\frac{1}{2}x}{\cos^2\frac{1}{2}x}}$$

$\cos^2\frac{1}{2}x$ نضرب البسط والمقام ب

$$= \frac{\cos^2\frac{1}{2}x - \sin^2\frac{1}{2}x}{\cos^2\frac{1}{2}x + \sin^2\frac{1}{2}x} = \cos x$$ (b)

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6 $$\frac{2\sin 2x\cos(-x)}{\sin 2x}$$

$$= 2\cos x$$ (a)

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7 $$\frac{1}{2}[\cos 30 - \cos 60] = \frac{1}{2}[\frac{\sqrt{3}}{2} - \frac{1}{2}]$$

$$= \frac{\sqrt{3}-1}{4}$$ (c)

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10 $$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

سنضرب المقام بالمرافق

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{(1-\cos x)^2}{(1-\cos^2 x)}} = \pm\sqrt{\frac{(1-\cos x)^2}{\sin^2 x}}$$

$$\tan\frac{x}{2} = \pm\frac{1-\cos x}{\sin x}$$

11 $$\frac{\sin 2x}{1+\cos 2x}\cdot\frac{\cos x}{1+\cos x}$$

$$= \frac{\sin 2x}{2\cos^2 x}\cdot\frac{\cos x}{1+\cos x}$$

$$= \frac{2\sin x\cos x}{2\cos x(1+\cos x)}$$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= \tan\left(\frac{x}{2}\right)$$

السؤال الرابع :

1 $$\sin 15^\circ\cos 15^\circ = \frac{1}{2}\sin 30^\circ = \frac{1}{4}$$ (b)

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2 $$\left(\frac{1}{2}\sin 45^\circ\right)^2$$

$$= \left(\frac{1}{2}\times\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{8}$$ (d)

12 $sec2x = \dfrac{1}{\cos 2x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

$\sin^2 x \cos^2 x$ نقسم كلاً من البسط والمقام على

$$sec2x = \frac{\frac{1}{\sin^2 x \cos^2 x}}{\frac{\cos^2 x}{\sin^2 x \cos^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x}}$$

$$= \frac{sec^2 x csc^2 x}{csc^2 x - sec^2 x}$$

$$= \frac{-sec^2 x csc^2 x}{sec^2 x - csc^2 x}$$ (b)

8 $(\sin 15° \sin 45°)^2$

$$= \left(\frac{1}{2}(\cos 60° - \cos 30°)\right)^2 = \left(\frac{1}{2}[\frac{1}{2} - \frac{\sqrt{3}}{2}]\right)^2$$

$$= \frac{1}{4} \cdot \frac{\left(1-\sqrt{3}\right)^2}{4} = \frac{1 - 2\sqrt{3} + 3}{16}$$

$$= \frac{1}{4} - \frac{\sqrt{3}}{8}$$ (b)

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9 $\dfrac{\sin(2x+x)}{\sin 2x} = \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\sin 2x}$

$$= \cos x + \frac{\cos 2x \sin x}{2 \sin x \cos x} = \cos x + \frac{2\cos^2 x - 1}{2 \cos x}$$ نوزع

$$= \cos x + \cos x - \frac{1}{2} secx = 2\cos x - \frac{1}{2} secx$$

(b)

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10 (b)

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11 $\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right)$

$$= \cos 2x$$ (b)

ورقة عمل (3)

السؤال الأول:

جد حل كلاً من المعادلات التالية في الفترة $[0, 2\pi)$:

a) $\frac{1-\cos x}{1+\cos x} = 3$

b) $\cos x \sin x - \sin^2 x = 0$

c) $\sin x - \cos x - \tan x = -1$

d) $\sin x - \cos x = \frac{1}{2}$

e) $\tan 3x + 1 = \sec 3x$

f) $4\cos^2\theta + 2\sin^2\theta = 3$

g) $3\sec^2\theta = 2\,csc\,\theta$

h) $4\sin x \cos x - 2\sqrt{3}\sin x - 2\cos x + \sqrt{3} = 0$

i) $\cot^2 x + 5\csc x = 5$

السؤال الثاني:

جد حل المعادلات التالية:

a) $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4$

b) $\sin^2 2x + \cos^2 x = 0$

c) $\sin x + \cos 2x - \sin 3x = 0$

d) $\sec^2 x - \tan^4 x = -1$

السؤال الثالث:

إذا علمت أن: $k\sin\theta = \sqrt{3}$, $k\cos\theta = 1$

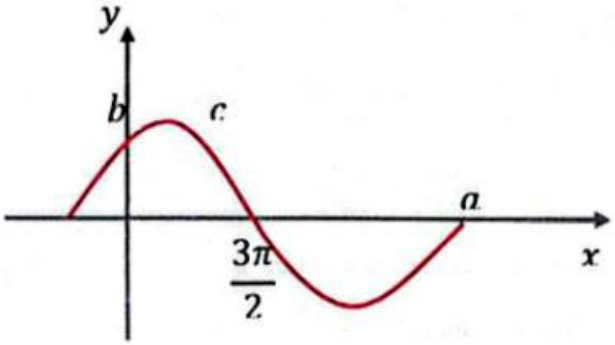
$k \in \mathbb{R} - \{0\}$.

a) جد قيمة (قيم θ) التي تحقق العلاقتين معاً.

b) جد قيم k الممكنة.

السؤال الرابع:

يمثل الشكل المجاور جزءاً من منحنى الاقتران $f(x) = \sin(x + k)$ جد كلاً مما يأتي:



a) قيمة a.

b) إحداثيا النقطة c.

c) أصغر قيمة لـ k.

d) إحداثيا النقطة b.

حل ورقة عمل (3)

حل السؤال الأول:

a) $\frac{1-\cos x}{1+\cos x} = 3$

الحل: (تبادلي)

$$3 + 3\cos x = 1 - \cos x$$

$$4\cos x = -2 \quad \Rightarrow \quad \cos x = \frac{-1}{2}$$

$$x \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$

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b) $\cos x \sin x - \sin^2 x = 0$

الحل: (عامل مشترك)

$$\sin x\,[\cos x - \sin x] = 0$$

$$\sin x = 0 \quad \Rightarrow \quad x = 0\ ,\ \pi$$

$$\cos x - \sin x = 0 \quad \Rightarrow \quad \sin x = \cos x$$

$$\tan x = 1 \quad \Rightarrow \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\left\{0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$$

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c) $\sin x - \cos x - \tan x = -1$

الحل:

$$\sin x - \cos x - \frac{\sin x}{\cos x} = -1$$

نضرب بـ $\cos x$ جميع الأطراف:

$$\sin x \cos x - \cos^2 x - \sin x = -\cos x$$

$$\underbrace{\sin x \cos x - \cos^2 x}_{\cos x \text{ مشترك}} \underbrace{- \sin x + \cos x}_{\text{سالب مشترك}} = 0$$

$$\cos x \underbrace{[\sin x - \cos x]}_{\text{مشترك}} - [\sin x - \cos x] = 0$$

$$(\sin x - \cos x)(\cos x - 1) = 0$$

$$\cos x = 1 \quad \Rightarrow \quad \boxed{x = 0}$$

$$\sin x = \cos x \quad \Rightarrow \quad \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$$

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d) $\sin x - \cos x = \frac{1}{2}$

الحل: نربع الطرفين

$$\sin^2 x - 2\sin x \cos x + \cos^2 x = \frac{1}{4}$$

$$1 - \sin 2x = \frac{1}{4} \quad \Rightarrow \quad \sin 2x = \frac{3}{4}$$

$2x = 0.85 \quad \Rightarrow \quad x \approx 0.425$ الأول

$2x = \pi - 0.85$ الثاني

$$x = \frac{\pi}{2} - 0.425 = 1.146 \quad \text{الثاني}$$

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e) $\tan 3x + 1 = \sec 3x$

الحل: نربع الطرفين:

$$\tan^2 3x + 2\tan 3x + 1 = \sec^2 3x$$

$$2\tan 3x + 1 = \underbrace{\sec^2 3x - \tan^2 3x}_{\text{متطابقة}}$$

$$2\tan 3x + 1 = 1 \quad \rightarrow \quad \tan 3x = 0$$

$$3x = 0\ ,\pi\ ,2\pi\ ,3\pi\ ,4\pi\ ,5\pi$$

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$$

✓ ✗ ✓ ✗ ✓ ✗

نتأكد من الحلول

$x = 0$ يحقق

$x = \frac{\pi}{3} \rightarrow 1 = -1$ مرفوض

$x = \frac{2\pi}{3}$ يحقق

$x = \pi \quad 1 = -1$ لا يحقق

$x = \frac{4\pi}{3}$ يحقق

$x = \frac{5\pi}{3}$ لا يحقق

$$\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$

h) $4\sin x\cos x - 2\sqrt{3}\sin x - 2\cos x + \sqrt{3} = 0$

الحل:

$$\underbrace{4\sin x\cos x - 2\sqrt{3}\sin x}_{2\sin x \text{ مشترك}} - \underbrace{2\cos x + \sqrt{3}}_{\text{سالب مشترك}} = 0$$

$$2\sin x\underbrace{[2\cos x - \sqrt{3}]}_{\text{مشترك}} - [2\cos x - \sqrt{3}] = 0$$

$$(2\sin x - 1)(2\cos x - \sqrt{3}) = 0$$

$$\sin x = \frac{1}{2}\begin{cases} \text{الأول } x = \frac{\pi}{6} \\ \text{الثاني } x = \frac{5\pi}{6} \end{cases}$$

$$\cos x = \frac{\sqrt{3}}{2}\begin{cases} \text{الأول } x = \frac{\pi}{6} \\ \text{الثاني } x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \end{cases}$$

$$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$$

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i) $\cot^2 x + 5\csc x = 5$

الحل:

$$(\csc^2 x - 1) + 5\csc x = 5$$
$$\csc^2 x + 5\csc x - 6 = 0$$
$$(\csc x + 6)(\csc x - 1) = 0$$

$$\csc x = -6 \quad \rightarrow \quad \sin x = -\frac{1}{6}$$

$$x = -0.167 \quad (\text{خارج المجال})$$

$$x_{\text{العام}} = -0.167 + 2\pi k$$

$$\rightarrow k = 1 \rightarrow x = 6.12$$

$$\csc x = 1 \rightarrow \sin x = 1 \rightarrow \boxed{x = \frac{\pi}{2}}$$

الحلول

$$\{\frac{\pi}{2}, \underbrace{6.12}_{1.95\pi \text{ تقريباً}}\}$$

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f) $4\cos^2\theta + 2\sin^2\theta = 3$

الحل:

$$4[1 - \sin^2\theta] + 2\sin^2\theta = 3$$
$$-4\sin^2\theta + 2\sin^2\theta + 4 = 3$$
$$-2\sin^2\theta = -1$$
$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{2}} \rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\sin\theta = -\frac{1}{\sqrt{2}} \rightarrow \theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$

نتأكد من الحلول

$x = \frac{\pi}{4} \rightarrow$ تحقق , $x = \frac{3\pi}{4} \rightarrow$ تحقق

$x = \frac{5\pi}{4} \rightarrow$ تحقق , $x = \frac{7\pi}{4} \rightarrow$ تحقق

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g) $3\sec^2\theta = 2\csc\theta$

الحل:

$$3\frac{1}{\cos^2\theta} = 2\frac{1}{\sin\theta}$$
$$2\cos^2\theta = 3\sin\theta$$
$$2\cos^2\theta - 3\sin\theta = 0$$
$$2[1 - \sin^2\theta] - 3\sin\theta = 0$$
$$-2\sin^2\theta - 3\sin\theta + 2 = 0$$
$$(*-1) \rightarrow \quad 2\sin^2\theta + 3\sin\theta - 2 = 0$$
$$(2\sin\theta - 1)(\sin\theta + 2) = 0$$

$\sin\theta = -2$ مرفوض

$$\sin\theta = \frac{1}{2}\begin{cases} \theta = \frac{\pi}{6} \\ \theta = \frac{5\pi}{6} \end{cases}$$

$$\theta \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$$

حل السؤال الثاني:

a) $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=4$

الحل:

$$\frac{(1+\sin x)^2+\cos^2 x}{\cos x\,(1+\sin x)}=4$$

$$1+2\sin x+\sin^2 x+\cos^2 x=4\cos x\,(1+\sin x)$$

$$2+2\sin x=4\cos x+4\sin x\cos x$$

$$2[1+\sin x]=4\cos x\,[1+\sin x]$$

$$[1+\sin x]-2\cos x\,[1+\sin x]=0$$

$$[1+\sin x][1-2\cos x]=0$$

$\cos x=\frac{1}{2}\ \rightarrow\ x=\frac{\pi}{3},\frac{5\pi}{3}$

$\sin x=-1\ \rightarrow\ x=\frac{3\pi}{2}$ مرفوضة (لا تحقق)

الحل العام

$$x=\frac{\pi}{3}+2k\pi$$

$$x=\frac{5\pi}{3}+2k\pi$$

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b) $\sin^2 2x+\cos^2 x=0$

الحل:

$$(2\sin x\cos x)^2+\cos^2 x=0$$

$$4\sin^2 x\cos^2 x+\cos^2 x=0$$

$$\cos^2 x\,[4\sin^2 x+1]=0$$

$\cos^2 x=0\ \rightarrow\ x=\frac{\pi}{2},\frac{3\pi}{2}$

$$\left.\begin{aligned}x&=\frac{\pi}{2}+2k\pi\\ x&=\frac{3\pi}{2}+2k\pi\end{aligned}\right\}$$

$4\sin^2 x+1=0$ لا يمكن

يمكن كتابة الحل $x=\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$

---

c) $\sin x+\cos 2x-\sin 3x=0$

الحل:

$$\sin x-\sin 3x=2\cos\left(\frac{4x}{2}\right)\sin\left(\frac{-2x}{2}\right)$$

$$=2\cos 2x\sin(-x)$$

أصبح شكل المعادلة:

$$-2\cos 2x\sin x+\cos 2x=0$$

$$\cos 2x\,[-2\sin x+1]=0$$

$\cos 2x=0\ \rightarrow\ 2x=\frac{\pi}{2}+k\pi$

$\therefore\ \boxed{x=\frac{\pi}{4}+\frac{k\pi}{2}}$

$\sin x=\frac{1}{2}\ \rightarrow\ \frac{\pi}{6}$ المرجع

$$x=\frac{\pi}{6}+2k\pi \qquad x=\frac{5\pi}{6}+2k\pi$$

---

d) $\sec^2 x-\tan^4 x=-1$

الحل:

$$1+\tan^2 x-\tan^4 x=-1$$

$$\tan^4 x-\tan^2 x-2=0$$

$$(\tan^2 x-2)(\tan^2 x+1)=0$$

$\tan^2 x=-1$ مستحيل

$\tan^2 x=2\ \rightarrow\ \tan x=\sqrt{2}\rightarrow x=0.955$

$\tan x=-\sqrt{2}\rightarrow x=-0.955$

الحل العام:

$$x=0.955+\pi k$$

$$x=-0.955+k\pi$$

حل السؤال الثالث:

$$k\sin\theta = \sqrt{3} \quad , \quad k \in \mathbb{R} - \{0\}$$
$$k\cos\theta = 1$$

a) جد قيمة (قيم θ) التي تحقق العلاقتين معاً.

الحل:

تقسم معادلة ① على معادلة ②:

$$\tan\theta = \sqrt{3}$$

الأول

$$\theta = 60° = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

الثالث

$$\theta = 60° + 180°$$
$$= 240°$$

$$\theta = \frac{4\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2l\pi$$
$$\theta = \frac{4\pi}{3} + 2l\pi \quad , \quad l \in \mathbb{R}$$

---

b) جد قيم k الممكنة.

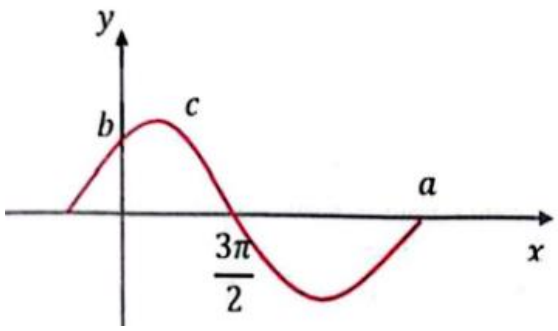
الحل:

نعوض الحلول

$$\theta = \frac{\pi}{3} \rightarrow k\sin\frac{\pi}{3} = \sqrt{3}$$
$$k = \sqrt{3} \times \frac{2}{\sqrt{3}} = 2$$

حل السؤال الرابع:

$$f(x) = \sin(x + k)$$



الحل:

نجد أولاً k: $\sin\left(\frac{2\pi}{3} + k\right) = 0$

أما الزاوية $= 2\pi, \pi, 0$

$0 = \frac{2\pi}{3} + k \rightarrow k = -\frac{2\pi}{3}$ مرفوضة

$k > 0$ حسب طبيعة b

حيث لو عوضنا $x = 0 \leftarrow$ القيمة سالبة

$$\pi = \frac{2\pi}{3} + k \rightarrow \boxed{k = \frac{\pi}{3}}$$

$$f(x) = \sin\left(x + \frac{\pi}{3}\right)$$

b نقطة تقطع محور $y \leftarrow x = 0$

$$f(0) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \rightarrow \left(0, \frac{\sqrt{3}}{2}\right)$$

c أعلى قيمة $\leftarrow y = 1$

$$1 = \sin\left(x + \frac{\pi}{3}\right) \rightarrow x + \frac{\pi}{3} = \frac{\pi}{2}$$
$$\rightarrow \boxed{x = \frac{\pi}{6}} \rightarrow c\left(\frac{\pi}{6}, 1\right)$$

لإيجاد a:

$$\sin\left(x + \frac{\pi}{3}\right) = 0 \rightarrow x + \frac{\pi}{3} = 2\pi$$
$$\rightarrow \boxed{x = \frac{5\pi}{3}} \rightarrow a\left(\frac{5\pi}{3}, 0\right)$$