3 Numbers and sequences

> 3.1 Counting and sequences

Worked example 1

Marcus counts back in eights starting at 50:

        50,   42,   34, ...

What is the first negative number Marcus says?

How do you know?

> linear sequence
>
> sequence
>
> term
>
> term-to-term rule

Six jumps of 8 is 48 so after counting back six lots of 8 Marcus says the number 2.	Another way to think about this question is:
	Each term is 2 more than a multiple of 8.
The next number Marcus says is –6.	10 is 2 more than a multiple of 8 and if you count back in 8s you get to 10, then 2 and then –6.

Answer: –6

Exercise 3.1

Focus

1   What is the next number in this sequence?

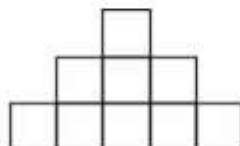   8, 6, 4, 2, 0, ...

   How do you know?

2 This is an 8 by 8 number grid.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

a Colour the multiples of 7.

b What do you notice?

_____

c If you continued the sequence would 100 be in it?

_____

3 This pattern makes a sequence.

You count the squares in each row of the staircase.

The sequence starts 1, 3, 5, ...

a Complete the table.

Row number	1	2	3	4	5	6	7
Number of squares in row	1	3	5				

b Continue the sequence to show your results.

1, 3, 5, —— , —— , —— , —— , —— , ——

c What is the term-to-term rule for the sequence?

_____

Practice

4 A sequence starts at 19.

9 is subtracted each time.

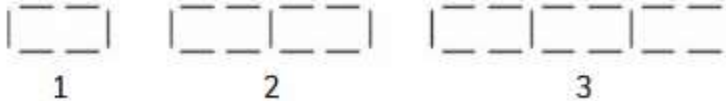What is the first number in the sequence that is less than zero?

_____

5 Zara counts back in steps of equal size.

Write the missing numbers in her sequence.

—— , —— , 61, 52, —— , 34

6 a Here are some patterns made from sticks.
   Draw the next pattern in the sequence.

1   2   3

b Complete the table.

Pattern number	Number of sticks
1	
2	
3	
4	

c Find the term-to-term rule.

_____

d How many sticks are in the 10th pattern?

_____

7 A sequence starts at 400 and 90 is added each time.
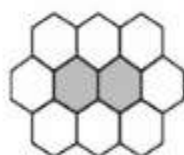
400,   490,   580,   670, ...

What is the first number in the sequence that is greater than 1000?
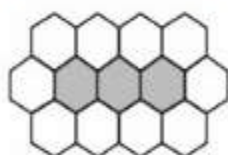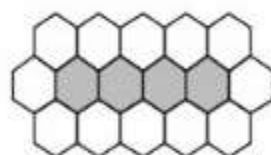
_____

Challenge

8 a Draw the next pattern in the sequence.

| 1 | 2 | 3 |

b Complete the table.

Pattern number	Number of hexagons
1	
2	
3	
4	

c Find the term-to-term rule.

_____

d How many hexagons are in the 8th pattern?

_____

9 Sofia counts in threes starting at 3.

$$3, \quad 6, \quad 9, \quad 12, ...$$

Zara counts in fives starting at 5.

$$5, \quad 10, \quad 15, \quad 20, ...$$

They both count on.

What is the **first** number greater than 100 that both Sofia and Zara say?

_____

Explain your answer.

_____

_____

10 Zara writes a sequence of five numbers.

The first number is 2.

The last number is 18.

Her rule is to add the same amount each time.

Write the missing numbers.

$$2, \text{_____} , \text{_____} , \text{_____} , 18$$

11 Arun writes a sequence of numbers.

His rule is to add the same amount each time.

Write the missing numbers.

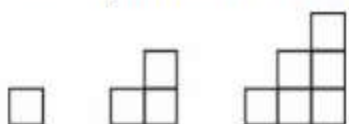$$-1, \text{_____} , \text{_____} , 14$$

> 3.2 Square and triangular numbers
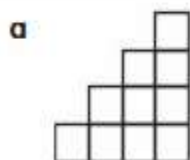
Worked example 2

Look at these patterns made from squares.

The sequence starts 1, 3, 6, ...

spatial pattern

square number

triangular number

a Draw the next term in the sequence.

b Write the next three numbers in the sequence.

---

a

Each term has an extra column that is one square taller.

b 10, 15, 21

The sequence 1, 3, 6, 10, 15, 21, ... is the sequence of triangular numbers.

Exercise 3.2

Focus

1 Draw the next term in this sequence.

What is the mathematical name for these numbers?

_____

2 Draw the next term in this sequence.

What is the mathematical name for these numbers?

_____
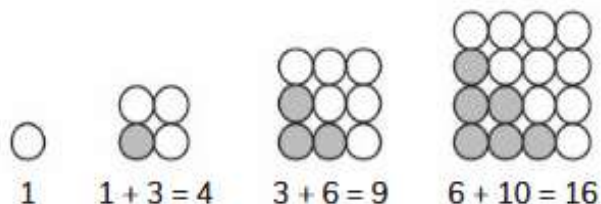
3 Write the next two terms in these sequences.

a 1, 4, 9, 16, 25, _____ , _____

b 1, 3, 6, 10, _____ , _____

Practice

4 This pattern is made by adding two consecutive triangular numbers.

1   1 + 3 = 4   3 + 6 = 9   6 + 10 = 16

> **Tip**
>
> Remember that consecutive means next to each other.
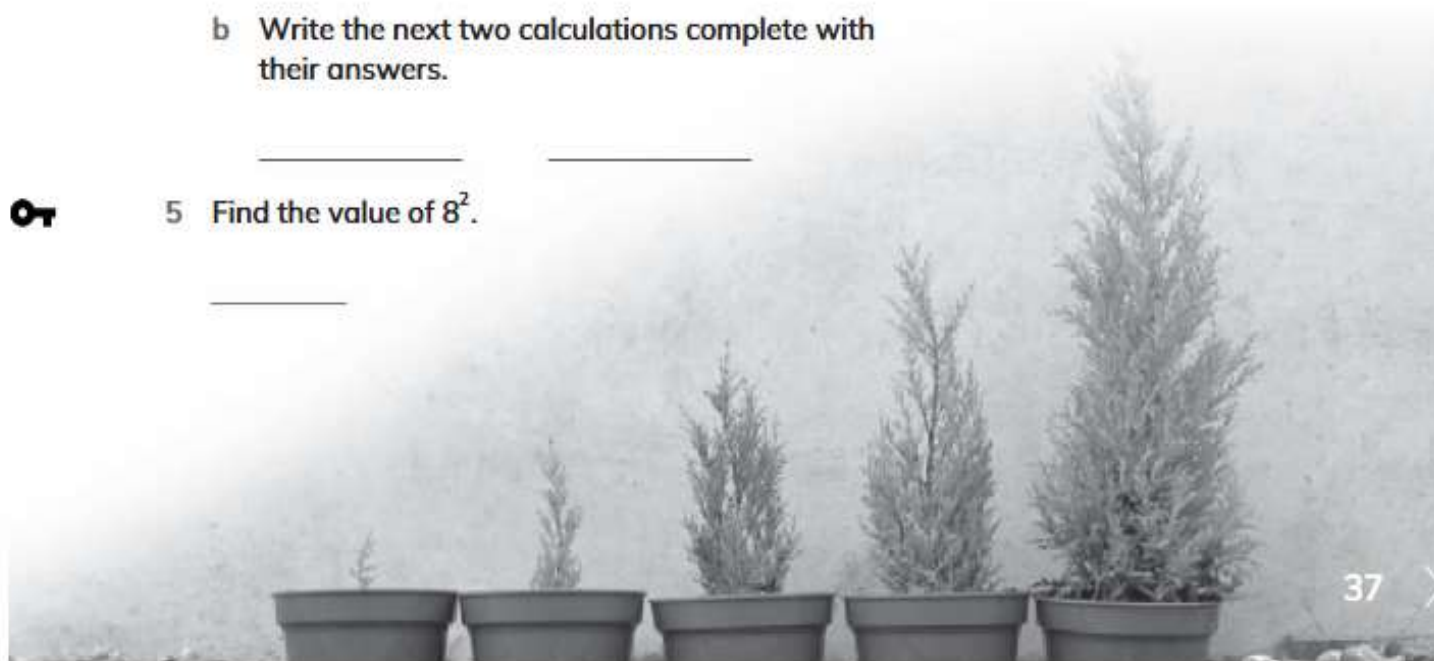
a Look at the total in each term and complete this sentence.

The terms in the sequence are the _____ numbers.

b Write the next two calculations complete with their answers.

_____   _____

5 Find the value of 8^2.

_____

6 Solve these number riddles.

a The number is:

- a square number
- a multiple of 3
- less than 25 _____

b The number is:

- a square number
- an even number
- a single digit number _____

c The number is:

- a square number
- a 2-digit number
- the sum of the digits is 13 _____

7 Zara has a lot of small squares all equal in size.

She uses the small squares to make big squares.

She can use 9 small squares to make a big square.
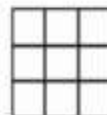
a Which of these numbers of squares could Zara use to make a big square?

46	64	14	4	24	100
60	66	18	81	9	90

_____

b Can Zara make a big square using 49 small squares?

Explain how you know.

_____

_____

Challenge

8 Write the missing numbers in the table.

Product of two numbers	1 more than the product	Equivalent square number
1 × 3 = 3	4	$2^2 = 4$
2 × 4 = 8	9	$3^2 = 9$
3 × 5 = 15	☐	$☐^2 = ☐$
4 × 6 = ☐	☐	$☐^2 = ☐$
☐ × ☐ = ☐	☐	$☐^2 = ☐$

Tip

Remember that each row continues the pattern.

9 Find two square numbers to make each of these calculations correct.

a ☐ + ☐ = 10   b ☐ + ☐ = 20

c ☐ + ☐ = 40   d ☐ + ☐ = 50

e ☐ + ☐ = 80   f ☐ + ☐ = 90

g ☐ + ☐ = 100

10 Find the 10th term in this sequence.

1, 3, 6, 10, 15, _____

2 a

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

   b The multiples form diagonal lines.

   c No

3 a 7, 9, 11, 13

   b 7, 9, 11, 13, 15, 17

   c Add 2

Practice

4 −8

5 79, 70, (61, 52,) 43

6 a

   b

Pattern number	Number of sticks
1	6
2	11
3	16
4	21

   c Add 5

   d 51 sticks

7 1030

Challenge

8 a

b
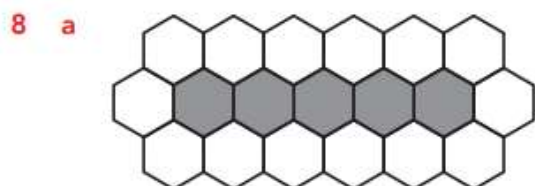
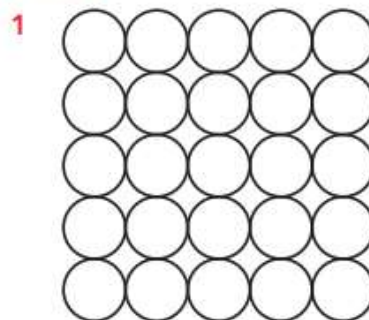| Pattern number	Number of hexagons
1	10
2	13
3	16
4	19

   c Add 3

   d 31 hexagons

9 105

The numbers that they both say must be multiples of both 3 and 5. The first number greater than 100 that is a multiple of both 3 and 5 is 105.
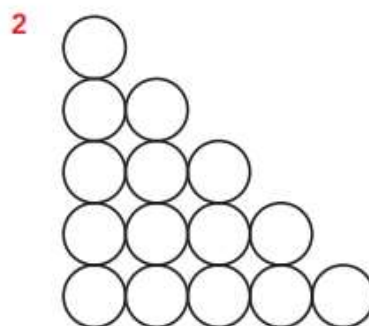
10 6, 10, 14

11 4 and 9

Exercise 3.2

Focus

1

square numbers

2

triangular numbers

3 a 36 and 49   **b** 15 and 21

Practice

4 a square

   b 10 + 15 = 25

      15 + 21 = 36

5 64

6 **a** 9   **b** 4   **c** 49

7 **a** 64, 4, 100, 81, 9

  b Yes. 49 is a square number so she can make a 7 by 7 square.

Challenge

8

$3 \times 5 = 15$	16	$4^2 = 16$
$4 \times 6 = 24$	25	$5^2 = 25$
$5 \times 7 = 35$	36	$6^2 = 36$

9 **a** $1 + 9 = 10$ **b** $4 + 16 = 20$

  c $4 + 36 = 40$

  d $25 + 25 = 50$ or $1 + 49 = 50$

  e $16 + 64 = 80$ **f** $9 + 81 = 90$

  g $36 + 64 = 100$

10 55