Chapter 2

Getting started

1 a C

b A

c A

2 Possible answers are:

a a^0

b $\frac{a^m}{a^n}$

c $a^m \times a^n$

d $(a^m)^n$

3 Example 1:

a Sign error

b $3x - x + 2$

Example 2:

a Multiplied both numerator and denominator by 3 instead of just the numerator. $\left(3 = \frac{3}{1} \text{ and not } \frac{3}{3}\right)$

b $\frac{3x + 12}{5}$

Example 3:

a Cancelled part of a term, but both x and 2 need to be divided by 2.

b $\frac{x + 2}{2}$ cannot be simplified further, but can be written as $\frac{x}{2} + 1$.

4 a There are different options, But in general, if you let one number be x, the consecutive number is $x + 1$. The sum of the numbers is $x + x + 1 = 2x + 1$. Any multiple of 2 is even, so if you add 1, it will be odd.

b Using the same argument $x + x + 1 + x + 2 = 3x + 3$. $3x$ can be odd or even (depending on the value of x) so the answer can be either odd or even.

Exercise 2.1

1 a $6xy$

b $7ab$

c xyz

d $2y^2$

e $4ab$

f $12xy$

g $5ab$

h yz^2

i $\frac{6}{x}$

j $\frac{4x}{2y} = \frac{2x}{y}$

k $\frac{x + 3}{4}$

l $\frac{m^3}{m^2} = m$

m $4x + 5y$

n $7a - 2b$

o $2x(x - 4)$

p $\frac{3(x + 1)}{2x}$

q $\frac{2(x + 4)}{3}$

r $\frac{4x}{6x} = \frac{2}{3}$

2 a $m + 13$

b $m + 5$

c $25 - m$

d m^3

e $\frac{m}{3} + 3$

f $4m - 2m = 2m$

3 a $x + 3$

b $x - 6$

c $10x$

d $-8 + x$

e $x + x^2$

f $x + 2x = 3x$

g $\frac{2x}{x + 4}$

4 a $(x - 10)$

b $\$\frac{x}{4}$

c \$15

5 a $m + 10$ years

b $m - 10$ years

c $\frac{m}{2}$ years

6 a $\$\frac{p}{3}$

b $\$\frac{p}{5}$, $\$\frac{p}{5}$ and $\$\frac{3p}{5}$

Exercise 2.2

1 a 9

b 30

c 10

d 27

e 18

f 7

g 16

h 36

i 4

j 6

k 6

l 30

m 5

n 2

o $\frac{1}{2}$

2 a 30

b 45

c 16

d 5

e 13

f 16

g 31

h 450

i 24

j 8

k 24

l 5

m $\frac{26}{3}$

n 10

o 4

p 3

q 6

r 225

s 12

t −10

u 129 600

3 a i $y = 0$

ii $y = 12$

iii $y = 16$

iv $y = 40$

v $y = 200$

b i $y = 1$

ii $y = 10$

iii $y = 13$

iv $y = 31$

v $y = 151$

c i $y = 100$

ii $y = 97$

iii $y = 96$

iv $y = 90$

v $y = 50$

d i $y = 0$

ii $y = 1.5$

iii $y = 2$

iv $y = 5$

v $y = 25$

e i $y = 0$

ii $y = 9$

iii $y = 16$

iv $y = 100$

v $y = 2500$

f i undefined

ii $y = 33.3$ (3 s.f.)

iii $y = 25$

iv $y = 10$

v $y = 2$

g i $y = 4$

ii $y = 10$

iii $y = 12$

iv $y = 24$

v $y = 104$

h i $y = -6$

ii $y = 0$

iii $y = 2$

iv $y = 14$

v $y = 94$

i i $y = 0$

ii $y = 81$

iii $y = 192$

iv $y = 3000$

v $y = 375\,000$

4 a $\$(3x + 2y)$

b i $18

ii $100

iii $350

5 a $P = 42$ cm

b $P = 8$ m

c $P = 60$ cm

d $P = 20$ cm

6 a i 43

ii 53

iii 71

iv 151

b They're all prime numbers.

c When $n = p$, $n^2 + n + p$ becomes $n(n + 2)$; in other words, it has factors n and $n + 2$, so is not prime.

What is the point of algebra?

Students' own work. There are many accessible examples that students could use, for example, conversion formulae, using algebra to find break even points (finance and economics), calculating doses of medicine based on person's mass or other factors, working out trajectories in sports (such as basketball or snooker), using BMI and other factors to work out health and fitness and so on.

Exercise 2.3

1 a $6x$, $4x$, x

b $-3y, \frac{3}{4}y, -5y$

c ab, $-4ba$

d $-2x$, $3x$

e $5a$, $6a$ and $5ab$, ab

f $-1xy$, $-yx$

2 a $8y$

b $7x$

c $13x$

d $22x$

e $5x$

f 0

g $-x$

h $-3y$

i $4x$

j $7xy$

k $4pq$

l $13xyz$

m $2x^2$

n $5y^2$

o $-y^2$

p $12ab^2$

q $5x^2y$

r $2xy^2$

3 a $5x + y$

b $4x + 2y$

c $7x$

d $4 + 4x$

e $6xy - 2y$

f $-x^2 + 2x$

g $-x + 4y$

h $3x + 3y$

i $8x + 6y$

j $8x - 2y$

k $14x^2 - 4x$

l $10x^2$

m $12xy - 2x$

n $8xy - 2xz$

o $-x^2 - 2y^2$

p $8x^2y - 2xy$

q $6xy - x$

r $6xy - 2$

4 a $2y - 8$

b $4x^2 - 5x$

c $7x + 4y$

d $y^2 + 5y - 7$

e $x^2 - 5x + 3$

f $x^2 + 5x - 7$

g $3xyz - 3xy + 2xz$

h $8xy - 10$

i $-3x^2 + 6x - 4$

5 a $P = 8x$

b $P = 4x + 14$

c $P = 6x + 3$

d $P = 5x + 4$

e $P = 12y - 6$

f $P = 8y^2 + 2y + 14$

g $P = 12y - 4$

h $P = 18x - 1$

Magic squares

1 No, the sum of numbers in a 3 × 3 magic square is always 3 times the centre number. In this case, that would be $12a + 15y$ for each row, column and diagonal and they do not all sum to that.

2 For example:

$a - c$	$a + b + c$	$a - b$
$a - b + c$	a	$a + b - c$
$a + b$	$a - b - c$	$a + c$

3 a

$x - 1$	$x + 1$	x
$x + 1$	x	$x - 1$
x	$x - 1$	$x + 1$

x	$x + 1$	$x - 1$
$x - 1$	x	$x + 1$
$x + 1$	$x - 1$	x

x	$x - 1$	$x + 1$
$x + 1$	x	$x - 1$
$x - 1$	$x + 1$	x

$x + 1$	$x - 1$	x
$x - 1$	x	$x + 1$
x	$x + 1$	$x - 1$

b Yes. Student's own magic squares or explanation.

Exercise 2.4

1 a $12x$
b $8y$
c $12m$
d $6xy$
e $8xy$
f $27xy$
g $24yz$
h $12xy$
i $8x^2y^2$
j $8x^2y$
k $27xy^2$
l $24xy^2$
m $8a^2b$
n $12ab^2c$
o $12a^2bc$
p $16a^2b^2c$
q $24abc$
r $72x^2y^2$

2 a $24x$
b $30x^2y$
c $12x^2y^2$
d x^3yz
e $48x$
f $24x^3y$
g $4x^2y^2$
h $12a^2bc$
i $60xy$
j $8xy$
k $9x^3y$
l $8x^3y^3$
m $42x^2y^2z^2$
n $56x^3y^2$
o $36x^2y^2z$
p $18x^4y^4$
q $54x^4y$
r $6x^3y^3$

3 a $5r$
b $4r$
c $3r$
d $6s$
e $7r$
f $2s$
g $\frac{s}{4}$
h $\frac{1}{4s}$
i $\frac{t}{2}$
j $6s$
k $\frac{1}{4}$
l $\frac{1}{9}$

4 a $4x$
b $6y$
c $\frac{4x}{y}$
d 8
e $\frac{7x^2}{y^2}$
f $3x$
g $\frac{x}{3}$
h $\frac{1}{4y}$
i $7y$
j $\frac{9y}{4}$
k $4xy$
l $\frac{4y}{x}$

5 a $\frac{ab}{6}$
b $\frac{a^2}{12}$
c $\frac{5a^2b}{6}$
d $\frac{10a}{3b}$
e $\frac{3ab}{8}$
f $\frac{25a^2}{4}$
g 2
h $\frac{a^2}{3}$
i $2ab$
j $\frac{8a}{3}$
k $\frac{1}{4}$
l a^2

Exercise 2.5

1 a $2x + 12$
b $3x + 6$
c $8x + 12$
d $10x - 60$
e $4x - 8$
f $6x - 9$
g $5a + 20$
h $24 + 6a$
i $9a + 18$
j $14c - 14d$
k $6c - 4d$
l $4c + 16d$
m $10x - 10y$
n $18x - 12y$
o $12y - 6x$
p $4s - 16t^2$
q $9t^2 - 9s$
r $28t + 7t^2$

2 a $2x^2 + 2xy$
b $3xy - 3y^2$
c $2x^2 + 4xy$
d $12x^2 - 8xy$
e $x^2y - xy^2$
f $12xy + 6y$
g $18ab - 8ab^2$
h $6a^2 - 4a^2b$
i $12a^2 - 12a^3$
j $36a - 8ab$
k $10b - 5ab$
l $12a - 3ab$
m $2x^2y^2 - 4x^3y$
n $12xy^2 - 8x^2y^2$
o $3x^2y^2 + 3xy^3$
p $2x^3y + x^2y^2$
q $81x^2 - 18x^3$
r $12xy^2 - 4x^2y^2$

3 a $A = x^2 + 7x$
b $A = 2x^3 - 2x$
c $A = 4x^2 - 4x$

Maths jokes

Student's own discussions, but could include puns, play on words, misinterpretation of concepts.

You could extend this by asking students to develop their own funny maths memes to share with the class.

Exercise 2.6

1 a $10 + 5x$
b $7y - 6$
c $4x - 8$
d $6x - 6$
e $2t^2 + 8t - 5$
f $4x + 1$
g $3x$
h $8x + 6$
i $6x + 9$
j $3h + 2$
k $8d + 6$
l $3y + xy - 4$
m $2x^2 + 8x - 4$
n $-4y^2 + 4xy + 8y$
o $10s - 12s^2$
p $6x^2 + 12x - 9$
q $-y^2 + 6y$
r $6x - 6$

2 a $6x + 154$
b $4x + 2$
c $7x + 26$
d 92
e $2x^2 + 16$
f $6p^2 + 10px$
g $24pq + 4p$
h $2xy + 4x$
i $-3x - 18xy$
j $21x - 12y - 2xy$
k $22x^2 - 7x^3$
l $x^2 - xy + 6x - 3y$
m $16s - 3st - 8$
n $2x^2$
o $4x^2 + 8xy$
p $2x^2 - 3x + 15$
q $9k - 17$
r $7xy + 9x$

Exercise 2.7

1 a $-30p - 60$
b $-15x - 21$
c $-20y - 1$
d $-3q + 36$
e $-24t + 84$
f $-12z + 6$
g $-6x - 15y$
h $-24p - 30q$
i $-27h + 54k$
j $-10h - 10k + 16j$
k $-8a + 12b + 24c - 16d$
l $-6x^2 - 36y^2 + 12y^3$

2 a $-5x - 8$
b $-5x + 12$
c $10x - 38$
d $-13f$
e $-36g + 37$
f $12y - 20$
g $-26x^2 - 76x$
h $-x^2 + 77x$
i $-9x^2 + 30x$

j $24q$
k $-42pq + 84p$
l $-48m + 48n$

3 a $12x - 6$
b $13x - 6$
c $-2x + 17$
d $x + 13$
e $23 - 7x$
f $10x - 8$
g $7x - 5$
h $x^2 - 5x + 8$
i $3x^2 - 7x + 2$
j $2x^2 + 3x + 6$
k $2x - 18$
l $6x^2 + 6x - 6$

Exercise 2.8

1 a x^8
b a^{10}
c y^2
d x^{13}
e y^9
f y^7
g y^6
h t^5
i $6x^7$
j $9y^6$
k $2m^4$
l $6s^7$
m $15x^3$
n $8x^7$
o $8z^7$
p $4x^7$

2 a x^2
b g^9
c y
d k^2
e s^4
f x^2
g $3x^2$
h $3p^3$
i $4y$
j $\frac{x}{2}$
k 3
l $3b$
m $\frac{1}{3x}$
n $4ab$
o 1

3 a a^4
b v^6
c f^{12}
d y^6
e $32x^{10}$
f $9c^4d^4$
g 1
h $125x^6$
i a^6b^6
j $x^{10}y^{20}$
k x^3y^{12}
l $16g^2h^4$
m $81x^8$
n x^4y^{24}
o 1

4 a $12x^6$
b $24x^3y$
c $4k^4$
d $\frac{x^2}{4}$
e $44x^3a^4b^2$
f $4x^3 + 28x$
g $4x^3 - x^5$
h x^2
i $\frac{7}{x^4}$
j $2x^2$
k $\frac{a^{12}}{b^6}$
l $\frac{x^4y^8}{16}$
m 1
n $8x^5$
o $2xy^3$

Exercise 2.9

1 a true
b false
c false
d false

2 a $\frac{1}{x^2}$
b $\frac{1}{y^3}$
c $\frac{1}{x^2y^2}$
d $\frac{2}{x^2}$
e $\frac{12}{x^3}$
f $\frac{7}{y^3}$
g $\frac{8x}{y^3}$
h $\frac{12}{x^3y^4}$

3 a b
b $\frac{6}{x^6}$
c $\frac{1}{3s^4}$
d $\frac{1}{h^{11}}$
e $\frac{1}{8x^6}$
f $\frac{1}{c^6}$
g x
h $\frac{1}{x^5}$
i $\frac{a^7}{b^5}$
j $\frac{x^{10}}{y^{12}}$
k $\frac{2y^{11}}{x^4}$
l 1
m $\frac{12n^{15}}{m^3}$

4 a $x = 4$
b $x = 5$
c $x = 2$
d $x = -3$
e $x = 3$
f $x = 3$
g $x = 2$
h $x = 4$

Exercise 2.10

1 a $x^{\frac{2}{3}}$
b $x^{\frac{7}{6}}$
c $\frac{1}{y^3}$
d $\frac{x^3}{y}$
e $a^{\frac{4}{7}}$
f $\frac{7b^2}{4}$
g $\frac{2}{x^2}$
h $\frac{3}{4k}$
i $3x^{\frac{7}{4}}$
j $\frac{1}{4x^{\frac{5}{2}}}$
k $\frac{s}{4}$
l $\frac{3x^{\frac{3}{4}}}{2}$
m $\frac{x}{8}$
n $\frac{1}{4x^{\frac{3}{2}}}$
o $x^{\frac{13}{12}}$
p $\frac{x^{\frac{1}{6}}}{y^{\frac{7}{6}}}$

2 a $x = 6$
b $x = \frac{1}{2}$
c $x = 16\,807$
d $x = 257$
e $x = 4$
f $x = 4$
g $x = 6$
h $x = 5$
i $x = 2$
j $x = -4$
k $x = \frac{1}{6}$
l $x = \frac{3}{4}$

Practice questions

1 **a** $n + 12$

b $2n - 4$

c $(nx)^2$

d $(n^2)^3$ or $(n^3)^2$ or n^6

2 **a** Any of $2n$, $4n$, $6n$,…

b $2n$ is always even because it is a multiple of 2. Every even number is 'next to' an odd number and $2n + 1$ is 'next to' $2n$.

c $p = 2n + 3$

d $2n + 1 + 2n + 3 = 4n + 4$ = even + even = even

3 **a** $p + 2q +$ r

b $8q$, $3r + 4q$

c Top brick = $2h + 2j + 2k$ = even + even + even = even

4 **a** $15xy + x$

b $5xy + 3y$

5 **a** a^2b

b $2x^6$

c $6x^4y^2$

d 1

e $4x^5y^3$

f $15x^2$

g x^3

h $16x^{-10}$ or $\frac{16}{x^{10}}$

i $\frac{27x^3}{64y^3}$

j $\frac{x^{19}}{3y^{12}}$

k $\frac{7p^6}{6q^{20}}$

6 0

7 **a** 10

b 10

c 10

8 7.35

9 **a** $8x - 4$

b $x^2 + 37xy$

10 **a** $m^2 - n^2$

b 0

11 **a** x^3

b $\frac{4}{x^2}$

c $\frac{1}{(2x-2)^3} = \frac{1}{8x^3 - 24x^2 + 24x - 8}$

12 **a** 3

b 3

c 3

13 18

14 **a** $15x$

b $9y^3$

c $4x$

15 **a** -3

b -3

c $\frac{1}{3}$

d $-\frac{1}{3}$

16 **a** pq

b $\frac{pq}{4}$

c p^3

17 **a** 4

b $\frac{15}{8}$

Practice questions worked solutions

1 **a** $n + 12$

b $2n - 4$

c $(nx)^3$

d $(n^2)^3$

2 **a** $2n$ (or $2n$ + any even number)

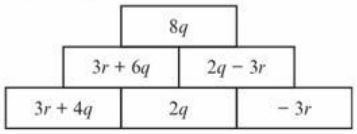
b $2n$ is always even and $2n + 1$ is one more than an even number. Whole numbers alternate
odd even odd even …
so $2n + 1$ must be odd.

c $p + 2 = 2n + 1 + 2$
$= 2n + 3$

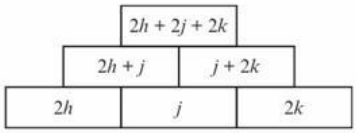
d $(2n + 1) + (2n + 3)$
$= 4n + 4$
$= 2(2n + 2)$, which is a multiple of 2 and hence even.

3 **a** $p + q + q + r$
$= p + 2q + r$

b



c



Top brick $= 2h + 2j + 2k$
$=$ three even numbers added together
$=$ even number

4 **a** $9xy + 3x + 6xy - 2x$
$= 3x - 2x + 9xy + 6xy$
$= x + 15xy$

b $6xy - xy + 3y$
$= 5xy + 3y$

5 **a** $\frac{a^{\cancel{3}2}b^{\cancel{4}1}}{{}^{1}\cancel{a}^{1}\cancel{b}^{3}} = a^2b$

b $2(x^3)^2 = 2x^6$

c $3x \times 2x^3y^2$
$= (3 \times 2) \times x \times x^3 \times y^2$
$= 6x^4y^2$

d $(4ax^2)^0 = 1$

e $4x^2y \times x^3y^2$
$= 4x^2 \times x^3 \times y \times y^2$
$= 4x^5y^3$

f $3x^{-4} \times 5x^6$
$= 3 \times 5 \times x^{-4} \times x^6$
$= 15 \times x^{-4+6}$
$= 15x^2$

g $\frac{3x^5}{7x^4} \div \frac{6x^{-6}}{14x^{-4}}$

$= \frac{{}^{1}\cancel{3}x^5}{{}_{1}\cancel{7}x^4} \times \frac{{}^{2}\cancel{14}x^{-4}}{{}_{2}\cancel{6}x^{-6}}$

$= x^{5-4-4-(-6)}$

$= x^3$

h $(4x^{-5})^2 = 4^2(x^{-5})^2$
$= 16x^{-10}$

i $\left(\frac{3x}{4y}\right)^3 = \frac{(3x)^3}{(4y)^3}$

$= \frac{27x^3}{64y^3}$

j $\frac{4x^{12}y^{-3}}{12x^{-7}y^9} = \frac{1}{3}x^{12-(-7)}y^{-3-9}$

$= \frac{1}{3}x^{19}y^{-12}$

$= \frac{x^{19}}{3y^{12}}$

k $\frac{{}^{7}\cancel{14}p^5q^{-4}}{{}_{6}\cancel{30}p^4q^4} \times \frac{{}^{1}\cancel{8}pq^{-7}}{{}_{1}\cancel{12}p^{-4}q^5}$

$= \frac{7}{6}p^{5+1-4-(-4)}q^{-4-4-7-5}$

$= \frac{7}{6}p^{10}q^{-20}$

$= \frac{7p^{10}}{6q^{20}}$

6 $7x^3y^2 \times (2x)^3 - (4x^3y)^2 - 4xy^2 \times 10x^5$
$= 7x^3y^2 \times 8x^3 - 16x^6y^2 - 40x^6y^2$
$= 56x^6y^2 - 16x^6y^2 - 40x^6y^2$
$= 0$

7 $x + 5 - (x - 5)$
$= x + 5 - x + 5$
$= 10$ for *all* values of x.
So, **a** 10 **b** 10 **c** 10

8 $s = \frac{1}{2}(u + v)t$

$= \frac{1}{2}\left(\frac{2}{5} + \frac{9}{2}\right) \times 3$

$= \frac{3}{2}\left(\frac{4 + 45}{10}\right) \times 3$

$= \frac{3 \times 49}{20}$

$= \frac{147}{20}$

9 **a** $5(x - 2) + 3(x + 2)$
$= 5x - 10 + 3x + 6$
$= 8x - 4$

b $5x(x + 7y) - 2x(2x - y)$
$= 5x^2 + 35xy - 4x^2 + 2xy$
$= x^2 + 37xy$

10 **a** $m(m - n) - n(n - m)$
$= m^2 - mn - n^2 + mn$
$= m^2 - n^2$

b $x(y - z) + y(z - x) + z(x - y)$
$= \cancel{xy} - xz + \cancel{yz} - \cancel{xy} + xz - \cancel{yz}$
$= 0$

11 a $x^5 \times x^{-2} = x^{5-2} = x^3$

b $\frac{{}^{4}\cancel{8}x^{\cancel{7}1}}{{}_{1}\cancel{2}x^{\cancel{4}2}} = \frac{4}{x^2}$

c $(2x-2)^{-3} = \frac{1}{(2x-2)^3}$

$= \frac{1}{2^3(x-1)}$

$= \frac{1}{8(x-1)}$

12 a $4^x = 43 \Rightarrow x = 3$

b $3^x - 5 = 22$

$3^x = 27$

$= 3^3$

$x = 3$

c $4 \times 6^p = 864$

$6^p = 216$

$= 6^3$

$p = 3$

13 $a^b - c^a + b^a$

$= 3^2 - (-1)^3 + 2^3$

$= 9 - (-1) + 8$

$= 17 + 1$

$= 18$

14 a $3x^{\frac{1}{2}} \times 5x^{\frac{1}{2}}$

$= 15x^{\frac{1}{2}+\frac{1}{2}}$

$= 15x$

b $(81y^6)^{\frac{1}{2}} = 81^{\frac{1}{2}}(y^6)^{\frac{1}{2}}$

$= \sqrt{81}y^3$

$= 9y^3$

c $(64x^3)^{\frac{1}{3}} = 64^{\frac{1}{3}}(x^3)^{\frac{1}{3}}$

$= \sqrt[3]{64}x^1$

$= 4x$

15 a $\left(\frac{1}{2}\right)^x = 8$

$(2^{-1})^x = 2^3$

$2^{-x} = 2^3$

$-x = 3$

So, $x = -3$

b $3^x = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$x = -3$

c $125^x = 5$

$(5^3)^x = 5$

$5^{3x} = 5^1$

$3x = 1$

So, $x = \frac{1}{3}$

d $125^x = \frac{1}{5}$

$(5^3)^x = 5^{-1}$

$5^{3x} = 5^{-1}$

$3x = -1$

$x = -\frac{1}{3}$

16 a $2^{x+y} = 2^x 2^y$

$= pq$

b $2^{x+y-2} = \frac{2^x 2^y}{2^2} = \frac{pq}{4}$

c $2^{3x} = (2^x)^3 = p^3$

17 a $n^{-1} = 2^{-2}$

$n^{-1} = (2^2)^{-1}$

$n = 2^2 = 4$

b $4^n = \left(\sqrt[4]{32}\right)^3$

$(2^2)^n = \left[(2^5)^{\frac{1}{4}}\right]^3$

$2^{2n} = 2^{\frac{5}{4}\times 3}$

$2n = \frac{15}{4}$

So $n = \frac{15}{8}$