

> Answers

The questions and example answers that appear in this resource were written by the author. In examination, the way marks would be awarded to answers like these may be different.

Chapter 1

Getting started

- 1
 - a Student answers will vary based on what they already know and feel confident doing.
 - b Some students will select the things they are less confident in, but other may select things they enjoy doing or are good at. Encourage them to say why they have made each selection.
- 2
 - a There are many possible answers for each value. For example, (a) could be 9^2 or $9(2 + 7)$ or $8 \times 10 + 1$. Let students use calculators to check that each other's clues work.
 - b Twenty-one thousand, eight hundred and thirty-seven
- 3
 - a 9^3
 - b 12^2
 - c 7^5
 - d $\left(\frac{1}{3}\right)^2$
 - e $\left(\frac{4}{3}\right)^0$
 - f $9^{\frac{1}{2}}$
 - g 14 000 010 019
- 4
 - a Any real-world measurement problems involve a level of approximation, as do problems where you have to work out if you have enough money, or have catered enough food, estimated times of arrivals, estimates for building materials and costs of doing different jobs.
 - b Encourage students to share ideas and discuss their own methods of deciding.
 - c Answers will vary, but could include that estimating allows you find errors and judge the size an answer should be, avoid mistakes due to button push or place value errors.

Exercise 1.1

- 1
 - a {3, 4, 6, 11, 16, 19, 25}
 - b {4, 6, 16}
 - c {3, 11, 19, 25}
 - d {-4, -1, 0, 3, 4, 6, 11, 16, 19, 25}
 - e {-4, -1}
 - f $\left\{\frac{1}{2}\right\}$
 - g {4, 16, 25}
 - h {3, 11, 19}
 - i {-4, -1, 0, $\frac{1}{2}$, 0.75, 6}
- 2
 - a {109, 111, 113, 115}
 - b Various, e.g. {2010, 2012, 2014, 2016} or {2020, 2022, 2024, 2026} etc.
 - c {995, 997, 999, 1001, 1003, 1005}
 - d {1, 4, 9, 16, 25}
 - e Various, e.g. {0.49, 0.48, 0.47, 0.46, 0.45} or {0.4, 0.3, 0.2, 0.1}
 - f Various, e.g. $\frac{1}{3}$, $\frac{3}{5}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{11}{20}$, $\frac{13}{20}$, $\frac{7}{10}$
- 3
 - a Even
 - b Even
 - c Odd
 - d Odd
 - e Even
 - f Even
- 4
 - a A perfect number is one where the sum of its factors, including 1, but excluding the number itself, is that number. 6 is perfect number because $1 + 2 + 3 = 6$.
 - b A palindromic number is a 'symmetrical' number like 16461 that remains the same when its digits are reversed.
 - c A narcissistic number is one that is the sum of its own digits each raised to the power of the number of digits, e.g. $371 = 3^3 + 7^3 + 1^3$.

Exercise 1.2

- 1 a $19 < 45$
b $12 + 18 = 30$
c $0.5 = \frac{1}{2}$
d $0.8 \neq 8.0$
e $-34 < 2 \times -16$
f $\therefore x = \sqrt{72}$
g $x \leq -45$
h π is approximately equal to 3.14
i $5.1 > 5.01$
j $3 + 4 \neq 3 \times 4$
k $12 - (-12) > 12$
l $(-12) + (-24) < 0$
m $12x$ is approximately equal to -40
- 2 a False
b True
c True
d True
e True
f True
g False
h True
i True
j True
k False
l False
m True
n False
- 3 Students' own discussions.

Exercise 1.3

- 1 a 2, 4, 6, 8, 10
b 3, 6, 9, 12, 15
c 5, 10, 15, 20, 25
d 8, 16, 24, 32, 40
e 9, 18, 27, 36, 45
f 10, 20, 30, 40, 50
g 12, 24, 36, 48, 60
h 100, 200, 300, 400, 500
- 2 a 29, 58, 87, 116, 145, 174, 203, 232, 261, 290
b 44, 88, 132, 176, 220, 264, 308, 352, 396, 440

- c 75, 150, 225, 300, 375, 450, 525, 600, 675, 750
d 114, 228, 342, 456, 570, 684, 798, 912, 1026, 1140
e 299, 598, 897, 1196, 1495, 1794, 2093, 2392, 2691, 2990
f 350, 700, 1050, 1400, 1750, 2100, 2450, 2800, 3150, 3500
g 1012, 2024, 3036, 4048, 5060, 6072, 7084, 8096, 9108, 10120
h 9123, 18246, 27369, 36492, 45615, 54738, 63861, 72984, 82107, 91230
- 3 a 32, 36, 40, 44, 48, 52
b 50, 100, 150, 200, 250, 300, 350
c 4100, 4200, 4300, 4400, 4500, 4600, 4700, 4800, 4900
- 4 576, 396, 792, 1164
- 5 816 and 1116

Exercise 1.4

- 1 a 10
b 40
c 12
d 9
e 385
f 66
- 2 No – the common multiples are infinite.

Exercise 1.5

- 1 a $F_4 = 1, 2, 4$
b $F_5 = 1, 5$
c $F_8 = 1, 2, 4, 8$
d $F_{11} = 1, 11$
e $F_{18} = 1, 2, 3, 6, 9, 18$
f $F_{12} = 1, 2, 3, 4, 6, 12$
g $F_{35} = 1, 5, 7, 35$
h $F_{40} = 1, 2, 4, 5, 8, 10, 20, 40$
i $F_{57} = 1, 3, 19, 57$
j $F_{90} = 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90$
k $F_{100} = 1, 2, 4, 5, 10, 20, 25, 50, 100$
l $F_{132} = 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132$
m $F_{160} = 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160$
n $F_{153} = 1, 3, 9, 17, 51, 153$
o $F_{360} = 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360$

- 2 a 4
b 45
c 14
d 22
e 8
- 3 a false
b true
c true
d true
e true
f true
g true
h false
- 4 The smallest factor is 1 and the largest factor is the number itself.

Exercise 1.6

- 1 a 3
b 8
c 5
d 14
e 4
f 2
g 22
h 6
- 2 a Any two from: 4, 6, 10, 14
b 12 and 18 are the only possible two, less than 20
- 3 1 because each prime number has only 1 and itself as factors.
- 4 18 m
- 5 20 students
- 6 150 bracelets

Why do mathematicians find prime numbers exciting?

- 1 a Every even integer greater than 2 can be written as the sum of two prime numbers.
b The weak conjecture is that every odd integer greater than 5 can be written as the sum of three odd prime numbers. Harald Helfgott's proof uses complicated mathematics to prove that this is correct. His proof is largely accepted by the mathematics community but they also acknowledge (as does he) that the strong

conjecture is much more difficult to prove and that the method used to prove the weak conjecture won't work for the strong one.

- 2 a The prime number theorem shows that prime numbers become less common as they get bigger using the rate at which prime numbers occur.
b Yes. Euclid (325–265BCE) proved there are infinitely many prime numbers. This proof is known as Euclid's theorem.
- 3 If you write prime backwards you get emirp. An emirp is a prime number that when you write it backwards gives you a different prime. For example, 17 and 71. The first few emirps are: 13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 149, 157.

Exercise 1.7

- 1 2
- 2 14
- 3 a 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28
b $6 = 3 + 3$, $8 = 3 + 5$,
 $9 = 2 + 7$, $10 = 5 + 5$,
 $12 = 5 + 7$, $14 = 3 + 11$,
 $15 = 2 + 13$, $16 = 5 + 11$,
 $18 = 5 + 13$, $20 = 3 + 17$,
 $21 = 2 + 19$, $22 = 5 + 17$,
 $24 = 5 + 19$ or $17 + 7$, $25 = 2 + 23$,
 $26 = 3 + 23$ or $13 + 13$, $27 =$ not possible,
 $28 = 5 + 23$
- 4 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73
- 5 149 is prime. Determined by trial division by all integers from 2 to $\sqrt{149}$

Exercise 1.8

- 1 a $30 = 2 \times 3 \times 5$
b $24 = 2 \times 2 \times 2 \times 3$
c $100 = 2 \times 2 \times 5 \times 5$
d $225 = 3 \times 3 \times 5 \times 5$
e $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
f $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$
g $650 = 2 \times 5 \times 5 \times 13$
h $1125 = 3 \times 3 \times 5 \times 5 \times 5$
i $756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$
j $9240 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 11$

Exercise 1.9

- 1 a 12
b 24
c 18
d 26
e 25
f 22
g 78
h 5
- 2 a 540
b 216
c 360
d 240
e 360
f 2850
g 270
h 360
- 3 a HCF = 36
LCM = 216
b HCF = 25
LCM = 200
c HCF = 5
LCM = 2280
d HCF = 12
LCM = 420
- 4 120 listeners
- 5 36 minutes
- 6 a 8
b 16
c 2^n

Exercise 1.10

- 1 a +\$100
b -25 km
c -10 marks
d +2 kg
e -1.5 kg
f 8000 m
g -10°C
h -24 m
i -\$2000
j +\$250
k -2 h
l +400 m

Exercise 1.11

- 1 a $2 < 8$
b $4 < 9$
c $12 > 3$
d $6 > -4$
e $-7 < 4$
f $-2 < 4$
g $-2 > -11$
h $-12 > -20$
i $-8 < 0$
j $-2 < 2$
k $-12 < -4$
l $-32 < -3$
m $0 > -3$
n $-3 < 11$
o $12 > -89$
p $-3 < 0$
- 2 a -12, -8, -1, 7, 10
b -10, -8, -4, -3, 4, 9
c -12, -11, -7, -5, 0, 7
d -94, -90, -83, -50, 0
- 3 a -4
b 10
c -14
d -3
e -2.7
f 5
g -6
h -6
i -27
j -4
k -4
l -5
- 4 a 1°C
b 1°C
c -3°C
d 12°C
e -3°C
- 5 \$28.50
- 6 a -\$420
b \$920
c -\$220
- 7 -11 m

$$5 \quad a \quad 324 = \underbrace{2 \times 2} \times \underbrace{3 \times 3} \times \underbrace{3 \times 3}$$

$$\sqrt{324} = 2 \times 3 \times 3$$

$$\sqrt{324} = 18$$

$$b \quad 225 = 3 \times 3 \times 5 \times 5$$

$$\sqrt{225} = 3 \times 5$$

$$\sqrt{225} = 15$$

$$c \quad 784 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{7 \times 7}$$

$$\sqrt{784} = 2 \times 2 \times 7$$

$$\sqrt{784} = 28$$

$$d \quad 2025 = \underbrace{3 \times 3} \times \underbrace{3 \times 3} \times \underbrace{5 \times 5}$$

$$\sqrt{2025} = 3 \times 3 \times 5$$

$$\sqrt{2025} = 45$$

$$e \quad 19\,600 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{5 \times 5} \times 7 \times 7$$

$$\sqrt{19\,600} = 2 \times 2 \times 5 \times 7$$

$$\sqrt{19\,600} = 140$$

$$f \quad 250\,000 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{5 \times 5} \times \underbrace{5 \times 5} \times \underbrace{5 \times 5}$$

$$\sqrt{250\,000} = 2 \times 2 \times 5 \times 5 \times 5$$

$$\sqrt{250\,000} = 500$$

$$6 \quad a \quad 27 = \underbrace{3 \times 3 \times 3}$$

$$\sqrt[3]{27} = 3$$

$$b \quad 729 = \underbrace{3 \times 3 \times 3} \times \underbrace{3 \times 3 \times 3}$$

$$\sqrt[3]{729} = 3 \times 3$$

$$\sqrt[3]{729} = 9$$

$$c \quad 2197 = \underbrace{13 \times 13 \times 13}$$

$$\sqrt[3]{2197} = 13$$

$$d \quad 1000 = \underbrace{2 \times 2 \times 2} \times \underbrace{5 \times 5 \times 5}$$

$$\sqrt[3]{1000} = 2 \times 5$$

$$\sqrt[3]{1000} = 10$$

$$e \quad 15\,625 = \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5}$$

$$\sqrt[3]{15\,625} = 5 \times 5$$

$$\sqrt[3]{15\,625} = 25$$

$$f \quad 32\,768 = \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2}$$

$$\sqrt[3]{32\,768} = 2 \times 2 \times 2 \times 2 \times 2$$

$$\sqrt[3]{32\,768} = 32$$

- 7 a 25
b 49
c 64
d 32
e 7
f 5
g 14
h 10
i 8
j 4
k 10
l 10
m 6
n 6
o 3
p $\frac{3}{2}$

- 8 a 10 cm
b 27 cm
c 41 mm
d 40 cm
- 9 a 31
b 17
c 65
d 17
e 68
f 24
g 730
h 82
i 33
j 129

Exercise 1.13

- 1 a 128
b 486
c 85
d 96
e 320
f 512
- 2 a $2^4 \times 3^4$ is greater by 1040
b $\sqrt[4]{625} \times 3^6$ is greater by 2877
- 3 a 2^6
b 3^5
c $2^4 \times 5^2$

- d $2^6 \times 5^2$
e 2^{14}
f $2^8 \times 3^4$
g 3^{10}
h 5^8

- 4 $25 = 5^2$
 $36 = 2^2 \times 3^2$
 $64 = 2^6$
The index is always even.

Exercise 1.14

- 1 a True
b False: 3^6
c True
d False: 8^6
e True
f True
g True
h False: 10^5
i False: 5^{-8}
j False: -2^8
k True
l False: -1

- 2 a 10^7
b 3^5
c 2^5
d 10^{-3}
e 10
f 12^0
g 3^{-7}
h 4^{-7}
i 3^{12}
j 5^{-4}
k 4^{-6}
l 4^0

- 3 a $\frac{5}{6}$ or 0.833
b $\frac{1}{36}$ or 0.0278
c $\frac{1}{2}$ or 0.5
d $\frac{1}{12}$ or 0.0833

- 4 a $\frac{1}{3}$
 b $\frac{1}{4}$
 c $\frac{1}{2}$
 d $\frac{1}{16}$
 e $\frac{1}{16}$
- 5 a 4^{-1}
 b 5^{-1}
 c 7^{-1}
 d 9^{-1}
 e $10\,000^{-1}$
 f 256^{-1}
 g 49^{-1}
 h 18^{-1}

- 6 a 5.0625
 b 1000
 c 2.25
 d 0.015 625
 e 36
 f 8
 g 13
 h 17

- 7 a 3^1
 b 3^2
 c 3^6
 d 3^{-3}
 e 3^{-1}
 f 3^0
 g 3^{-5}
 h $-(3^2)$

Exercise 1.15

- 1 a $\sqrt{25}$
 b $\sqrt[3]{3}$
 c $\sqrt{40}$
 d $\sqrt{6}$
 e $\sqrt[8]{3}$
 f $(\sqrt[4]{2})^3$
 g $(\sqrt[3]{12})^2$
 h $(\sqrt[9]{5})^2$

- 2 a $5^{\frac{1}{2}}$
 b $8^{\frac{1}{3}}$
 c $13^{\frac{1}{3}}$
 d $11^{\frac{1}{4}}$
 e $9^{\frac{2}{3}}$
 f $6^{\frac{4}{3}}$
 g $32^{\frac{1}{4}}$
 h $2(12^{\frac{2}{3}})$

- 3 a 5
 b 3
 c 4
 d 8
 e 36
 f 0.5
 g 6.78
 h 0.0016
 i 0.5
 j 16
 k 36
 l 64

- 4 a $c = 70 \times (\sqrt[4]{m})^3$
 b 251.40 calories
 c 41 622.25 calories

Exercise 1.16

- 1 a $(4 + 7) \times 3$
 $= 11 \times 3$
 $= 33$
 b $(20 - 4) \div 4$
 $= 16 \div 4$
 $= 4$
 c $50 \div (20 + 5)$
 $= 50 \div 25$
 $= 2$
 d $6 \times (2 + 9)$
 $= 6 \times 11$
 $= 66$
 e $(4 + 7) \times 4$
 $= 11 \times 4$
 $= 44$
 f $(100 - 40) \times 3$
 $= 60 \times 3$
 $= 180$

$$\begin{aligned} \text{g} \quad & 16 + (25 \div 5) \\ & = 16 + 5 \\ & = 21 \end{aligned}$$

$$\begin{aligned} \text{h} \quad & 19 - (12 + 2) \\ & = 19 - 14 \\ & = 5 \end{aligned}$$

$$\begin{aligned} \text{i} \quad & 40 \div (12 - 4) \\ & = 40 \div 8 \\ & = 5 \end{aligned}$$

$$\begin{aligned} \text{j} \quad & 100 \div (4 + 16) \\ & = 100 \div 20 \\ & = 5 \end{aligned}$$

$$\begin{aligned} \text{k} \quad & 121 \div (33 \div 3) \\ & = 121 \div 11 \\ & = 11 \end{aligned}$$

$$\begin{aligned} \text{l} \quad & 15 \times (15 - 15) \\ & = 15 \times 0 \\ & = 0 \end{aligned}$$

$$2 \quad \text{a} \quad 108$$

$$\text{b} \quad 72$$

$$\text{c} \quad 3$$

$$\text{d} \quad 10$$

$$\text{e} \quad 32$$

$$\text{f} \quad 9$$

$$\text{g} \quad 5$$

$$\text{h} \quad 1$$

$$\text{i} \quad 140$$

$$3 \quad \text{a} \quad \begin{aligned} & 5 \times 10 + 3 \\ & = 50 + 3 \\ & = 53 \end{aligned}$$

$$\begin{aligned} \text{b} \quad & 5 \times (10 + 3) \\ & = 5 \times 13 \\ & = 65 \end{aligned}$$

$$\begin{aligned} \text{c} \quad & 2 + 10 \times 3 \\ & = 2 + 30 \\ & = 32 \end{aligned}$$

$$\begin{aligned} \text{d} \quad & (2 + 10) \times 3 \\ & = 12 \times 3 \\ & = 36 \end{aligned}$$

$$\begin{aligned} \text{e} \quad & 23 + 7 \times 2 \\ & = 23 + 14 \\ & = 37 \end{aligned}$$

$$\begin{aligned} \text{f} \quad & 6 \times 2 \div (3 + 3) \\ & = 12 \div 6 \\ & = 2 \end{aligned}$$

$$\begin{aligned} \text{g} \quad & \frac{15 - 5}{2 \times 5} \\ & = \frac{10}{10} \\ & = 1 \end{aligned}$$

$$\begin{aligned} \text{h} \quad & (17 + 1) \div 9 + 2 \\ & = 18 \div 9 + 2 \\ & = 2 + 2 \\ & = 4 \end{aligned}$$

$$\begin{aligned} \text{i} \quad & \frac{16 - 4}{4 - 1} \\ & = \frac{12}{3} \\ & = 4 \end{aligned}$$

$$\begin{aligned} \text{j} \quad & 17 + 3 \times 21 \\ & = 63 + 17 \\ & = 80 \end{aligned}$$

$$\begin{aligned} \text{k} \quad & 48 - (2 + 3) \times 2 \\ & = 48 - 5 \times 2 \\ & = 48 - 10 \\ & = 38 \end{aligned}$$

$$\begin{aligned} \text{l} \quad & 12 \times 4 - 4 \times 8 \\ & = 48 - 32 \\ & = 16 \end{aligned}$$

$$\begin{aligned} \text{m} \quad & 15 + 30 \div 3 + 6 \\ & = 15 + 10 + 6 \\ & = 31 \end{aligned}$$

$$\begin{aligned} \text{n} \quad & 20 - 6 \div 3 + 3 \\ & = 20 - 2 + 3 \\ & = 21 \end{aligned}$$

$$\begin{aligned} \text{o} \quad & 10 - 4 \times 2 \div 2 \\ & = 10 - 4 \div 1 \\ & = 10 - 4 \\ & = 6 \end{aligned}$$

$$4 \quad \text{a} \quad 7$$

$$\text{b} \quad 7$$

$$\text{c} \quad 3$$

$$\text{d} \quad 0$$

$$\text{e} \quad 3$$

$$\text{f} \quad 10$$

- 5 a 13
b 8
c 58
d 192
e 12 000
f 1660
g 260
h 868
- 6 a 18
b 3
c 3
d 8
e 4
f 9
- 7 a False
b True
c False
d True
- 8 a $3 \times (4 + 6) = 30$
b $(25 - 15) \times 9 = 90$
c $(40 - 10) \times 3 = 90$
d $(14 - 9) \times 2 = 10$
e $(12 + 3) \div 5 = 3$
f $(19 - 9) \times 15 = 150$
g $(10 + 10) \div (6 - 2) = 5$
h $(3 + 8) \times (15 - 9) = 66$
i $(9 - 4) \times (7 + 2) = 45$
j $(10 - 4) \times 5 = 30$
k $6 \div (3 + 3) \times 5 = 5$
l BODMAS means that brackets are not needed.
m $(1 + 4) \times (20 \div 5) = 20$
n $(8 + 5 - 3) \times 2 = 20$
o $36 \div (3 \times 3 - 3) = 6$
p $3 \times (4 - 2) \div 6 = 1$
q $(40 \div 4) + 1 = 11$
r BODMAS means that brackets are not needed.
- 9 a $2 - 10 \div 5 = 0$
b $13 - 18 \div 9 = 11$
c $8 \div (16 - 14) - 3 = 1$
d $(9 + 5) - (6 - 4) = 12$
or $(9 + 5) - (12 - 4) = 6$

Exercise 1.17

- 1 a -10
b 8.86
c 13
d 29
e -22
f 8.75
g 20
h 0
i 4
j 70
k 12
l 20
m 8
n 15
o 20
- 2 a Correct
b Incorrect = 608
c Correct
d Correct
e Incorrect = 368
f Incorrect = 10
- 3 a $12 \div (28 - 24) = 3$
b $84 - 10 \times 8 = 4$
c $3 + 7(0.7 + 1.3) = 17$
d $23 \times 11 - 22 \times 11 = 11$
e $40 \div 5 \div (7 - 5) = 4$
f $9 + 15 \div (3 + 2) = 12$
- 4 a 0.5
b 2
c 0.183
d 0.5
e $\frac{1}{3}$ is approximately equal to 0.333 (3 s.f.)
f 1
g 2
h $\frac{2}{3}$ is approximately equal to 0.667 (3 s.f.)
- 5 Correct to 3 significant figures
a 0.0112
b 0.0950
c -0.317

- f** $6 \times \sqrt{20} = 6 \times 4.5$ (approximate root between $\sqrt{16}$ and $\sqrt{25}$) = 27, so sensible
- 2 a** $\frac{23.6}{6.3}$ is approximately equal to $\frac{24}{6} = 4$
- b** $\frac{4}{0.09 \times 4}$ is approximately equal to $\frac{4}{0.36}$ is approximately equal to 11
- c** $\frac{7 \times 0.5}{9}$ is approximately equal to $\frac{3.5}{9}$ is approximately equal to 0.39
- d** $\frac{5 \times 6}{2.5 + 1}$ is approximately equal to $\frac{30}{3.5}$ is approximately equal to 8.6
- e** $\frac{\sqrt{49}}{2.5 + 4}$ is approximately equal to $\frac{7}{6.5}$ is approximately equal to 1
- f** $(0.5 + 2)(6.5 - 2)$ is approximately equal to $(2.5)(4.5)$ is approximately equal to 11.3
- g** $\frac{24 + 20}{5 + 6}$ is approximately equal to $\frac{44}{11} = 4$
- h** $\frac{110 - 45}{19 - 14}$ is approximately equal to $\frac{65}{5} = 13$
- i** $3^2 \times \sqrt{49}$ is approximately equal to $9 \times 7 = 63$
- j** $\sqrt{224 \times 45}$ is approximately equal to $\sqrt{10080}$ is approximately equal to 100
- k** $\sqrt{9} \times \sqrt{100}$ is approximately equal to $3 \times 10 = 30$
- l** $4^3 \times 2^4$ is approximately equal to $64 \times 16 = 1024$

3 Answers given to 1 d.p.

- a** 3.7
b 12.7
c 0.4
d 8.0
e 1.0
f 10.8
g 4.2
h 11.7
i 44.4
j 100.5
k 30.4
l 898.2

Making decisions about accuracy

- 1 a** Whole numbers
b 2 d.p.
c Millions
d 4 d.p.
e 3 s.f.
- 2 a** Zaf changed decimals to fractions to easily divide by 2. Marwan cancelled before rounding to have fewer numbers to deal with.
b Once you have rounded, you are calculating exact values, so even if 2 and 3 are rounded values, $2 + 3$ is equal to 5, not approximately equal to 5.
- 3** Possible examples:
a Overestimate the cost of buying several items to make sure you definitely have enough money
b Underestimate the size of a doorway to make sure you have enough room to move furniture though it.

Practice questions

- 1** $49 - 30 = 19$
- 2** 9 and -4 or -9 and 4
- 3** 15
- 4** 216 216
- 5** 735
 736
 737
 738
 739
 741
 742
 743
 744
- 6** $1080 = 2^3 \times 3^3 \times 5$
 1080 is not a cube number. Not all the factors are powers with indices that are multiples of 3.
- 7 a** 33 and 61
b 26 and 45
- 8 a** 32
b 340
c 25

7 a $2013 = 3 \times 11 \times 61$
 $= 33 \times 61$
 $33 + 61 = 94$ so the numbers are 33 and 61.

b $1170 = 2 \times 3 \times 3 \times 5 \times 13$
 $= (3 \times 3 \times 5) \times (2 \times 13)$
 $= 45 \times 26$
 $45 - 26 = 19$ so the numbers are 45 and 26.

8 a $12 + 20 = 32$

b $4 \times 85 = 340$

c $11 \times 2 + (15 - 6) - 6$
 $= 22 + 9 - 6$
 $= 25$

d $-15 - (-48)$
 $= -15 + 48$
 $= 33$

e $-3 \times (-11) + (-24)$
 $= 33 - 24$
 $= 9$

f $(-4)^3 + 16$
 $= -64 + 16$
 $= -48$

9 a $5 + 7 - 3 - 8 = 1$

b $(5 - 3^2) \times 6 + 8 \div (-2)$
 $= -4 \times 6 + (-4)$
 $= -24 - 4$
 $= -28$

10 $(7 + 14) \div (4 - 1) \times 2 = 14$

11 1.16 (to 3 s.f.)

12 a is approximately equal to
 $\frac{5 - 5^2}{\sqrt{25}} = \frac{5 - 25}{5} = \frac{-20}{5} = -4$

b Calculator answer = $-4.276\,348\,739 \dots$
Difference = $0.276\,348\,739 \dots$
 $= 0.276$ (to 3 s.f.)

13 A $4 \times (4 + 16) = 4 \times 20 = 80$

B $= \frac{64}{16} + 4 = 4 + 4 = 8$

C $= \frac{16 - 4}{4} = 3$

D $= 16 - 16 \times 4 + 1$
 $= 16 - 64 + 1$
 $= -47$

The order is D, C, B, A.

14 a $\sqrt{98} + \sqrt{72} = \sqrt{49 \times 2} + \sqrt{36 \times 2}$
 $= 7\sqrt{2} + 6\sqrt{2}$
 $= 13\sqrt{2}$

b $(3^{-2} + 2^{-3}) \times 216^{\frac{2}{3}}$
 $= \left(\frac{1}{3^2} + \frac{1}{2^3}\right) \times (\sqrt[3]{216})^2$
 $= \left(\frac{1}{9} + \frac{1}{8}\right) \times 6^2$
 $= \frac{17}{72} \times 36 = \frac{17}{2}$

c $((\sqrt{2})^2 + 23)^{\frac{1}{2}}$
 $= \sqrt{(2 + 23)}$
 $= \sqrt{25}$
 $= 5$

d $\left(\frac{36}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{36}{25}}\right)^3$
 $= \left(\frac{6}{5}\right)^3$
 $= \frac{216}{125}$

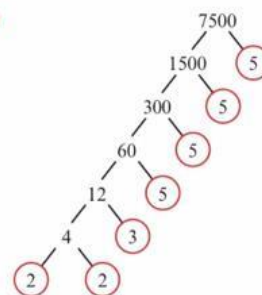
e $\left(\frac{16}{81}\right)^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{\frac{16}{81}}}$
 $= \frac{1}{\left(\frac{2}{3}\right)}$
 $= \frac{3}{2}$

15 a $60 = 2 \times 2 \times 3 \times 5$
 $36 = 2 \times 2 \times 3 \times 3$

b $2 \times 2 \times 3 \times 3 \times 5 = 180$

c 180 days after 1 March 2023 is 28 August 2023.

16



$7500 = 2^2 \times 3^1 \times 5^4$
 $= B A D$

d 33

e 9

f -48

9 a $5 \times 7 - 3 \times 8 = 11$

b $(5 - 3^2) \times 6 + 8 \div (-2) = -28$

10 $(7 + 14) \div (4 - 1) \times 2 = 14$

11 1.16

12 a -4

b 0.276 to 3 s.f.

13 D, C, B, A

14 a $\sqrt{338}$

b $\frac{17}{2}$

c 5

d $\frac{216}{125}$

e $\frac{3}{2}$

15 a $60 = 2^2 \times 3 \times 5$

$36 = 2^2 \times 3^2$

b $\text{LCM} = 2^2 \times 3^2 \times 5$
 $= 180$

c 28 August 2023

16 BAD

Practice questions worked solutions

1 The prime numbers smaller than 20 are:
2, 3, 5, 7, 11, 13, 17, 19

Sum of the three largest prime numbers
smaller than 20
 $= 13 + 17 + 19$
 $= 49$

Product of the three smallest prime numbers
 $= 2 \times 3 \times 5$
 $= 30$

Difference $= 49 - 30$
 $= 19$

2 Product $= -36$, which is negative \Rightarrow one number is positive and the other is negative.

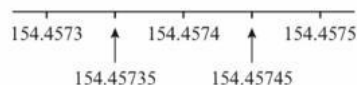
Factor pairs of 36: 1×36
 2×18
 3×12
 4×9
 6×6

You can make a difference of 13 with either 9 and -4 or -9 and 4.

3 The number is one fifteenth of its own square.
 \Rightarrow The number must be multiplied by 15 to square it.
 \Rightarrow The number is 15.

4 $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 \times 13$
 $= 216\,216$

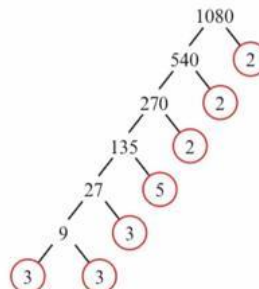
5 Look at 154.4574 on a number line and you will see that the number 154.45ABC must lie between 154.45735 and 154.45744. (154.45745 rounds up to 154.45735.)



So, A must be 7. The possibilities are:

A	B	C
7	3	5
7	3	6
7	3	7
7	3	8
7	3	9
7	4	1
7	4	2
7	4	3
7	4	4

6



$1080 = 2^3 \times 3^3 \times 5$

The power of 5 is not a multiple of 3 so 1080 is **not** a cube number.