Greek Orthodox Patriarchate School
International - Hanina

MATH BOOKLET

NAME: _____

GRADE: _____

- Write a positive integer as a product of prime factors.
- Use prime factors to find the Highest common factor (HCF) and a lowest common multiple.

TEACHER: YARA ALAWADEH

Any Integer bigger than 1:

- Is a prime number, or
- Can be written as a product of prime numbers.

Examples

46 = 2×23

47 is prime

50= 2×5×5

48= 2×2×2×2×3

You can use a Factor tree to write an integer as a product of its Prime factors.

A factor tree is a diagram we use in math to break down a number into its prime factors (the prime numbers that multiply together to make the original number).

How to do a factor tree:

1. Start with your number.
2. Choose any two factors of that number (numbers that multiply to it).
    - Example: for 24, you might start with 6×4 or 12×2.
3. Break down each factor further, unless it's already a prime number.
4. Continue until all the "leaves" of the tree are prime numbers.
5. The result is the prime factorization of the number.

Factor Tree for 120

1. Start with **120**.
2. Split into factors: **12 × 10** (you can choose other factors too; the tree can look slightly different).
3. Factor 12: **3 × 4**
   Factor 4: **2 × 2**
4. Factor 10: **2 × 5**

So all the prime factors are: **2, 2, 2, 3, 5**

✅ Prime factorization: $120=2^3 \times 3 \times 5$

Factor Tree for 75

1. Start with **75**.
2. Split into factors: **5 × 15**
3. Factor 15: **3 × 5**

Prime factors: **3, 5, 5**

✅ Prime factorization:

$75= 3 \times 5^2$

Now we're connecting factor trees to **HCF (Highest Common Factor)** and **LCM (Lowest Common Multiple)**. Factor trees are very useful because they give you **the prime factorization** of numbers, which makes finding HCF and LCM systematic. Let me explain step by step.

Step 1: Get the prime factors using factor trees

From your earlier examples:

- $120 = 2^3 \times 3 \times 5$
- $75 = 3 \times 5^2$

Step 2: Finding HCF

- HCF is the product of the prime factors common to both numbers, with the smallest powers.
- Compare the prime factors:

Prime	120	75	HCF
2	2^3	—	—
3	3^1	3^1	3^1
5	5^1	5^2	5^1

Multiply the common primes: $HCF = 3 \times 5 = 15$

Step 3: Finding LCM

- LCM is the product of all prime factors present in either number, with the largest powers.
- Compare the prime factors:

Prime	120	75	LCM
2	2^3	—	2^3
3	3^1	3^1	3^1
5	5^1	5^2	5^2

Multiply all largest powers: **LCM = $2^3 \times 3 \times 5^2 = 600$**

✅ Summary

- **Factor trees → prime factorization**
- **HCF:** take **common primes with smallest powers**
- **LCM:** take **all primes with largest powers**

There are a few simple ways to find the prime factors of a large number without getting lost.

Step 1: Start with the smallest prime (2)

- If the number is even, divide by 2 repeatedly until it's odd.
- Example: 2400÷2=1200÷2=600÷2=300÷2=150÷2=75

Step 2: Move to the next prime (3)

- Check if the remaining number is divisible by 3.
- Divide repeatedly by 3 if possible.

Step 3: Continue with next primes (5, 7, 11, …)

- Test divisibility by primes in order.
- Stop when the remaining number is 1 or a prime itself.

Step 4: Optional trick for checking divisibility

- **2:** even numbers
- **3:** sum of digits divisible by 3
- **5:** ends with 0 or 5
- **7, 11, 13, …:** smaller prime checks or trial division

Step 5: Write as a product of primes

- Multiply all the prime numbers you divided by.
- Example: $2400=2^5\times3\times5^2$

Here are some questions.

- Write these numbers as a product of prime numbers.

a. 108	b. 70^3
c. 8712	d. 1001

- Use a factor tree to write 132 and 150 as a product of prime numbers.
  132×150=19800. Use this fact to write 19800 as a product of prime numbers.

- Work out the following.

a. $2\times3\times7$	b. $2^2\times3^2\times7^2$

- Write each square number as a product of its prime factors.

a. 576	b. 2401

Prime Factorization of a Square Number:

- A square number is made by multiplying a number by itself.
- When you write it as prime factors, **all the powers are even**.

  Example:
- $16=4^2$
- Prime factorization of 4: 2×2
- Prime factorization of 16: $2^2\times2^2=2^4$

  Simple rule: **All exponents in a square number's prime factorization are even.**

- When a square number is written as a product of prime numbers, what can you say about the factors?

- If $40=2\times2\times2\times5$ and $28=2\times2\times7$

  Use these facts to find

a. the HCF of 40 and 28

b. The LCM of 40 and 28

- Write 396 as a product of prime numbers. Then write 168 as a product of prime numbers.

  a. Find the HCF of 396 and 168

  b. Find the LCM of 396 and 168

- Write 343 as a product of prime numbers. Then write 546 as a product of prime numbers.

- a. Find the HCF of 343 and 546

- b. Find the LCM of 343 and 546

- The HCF of two numbers is 6. The LCM of the two numbers is 72. What are the two numbers?

- Read Rebeca's exam answers. Mark as correct or incorrect or incomplete. Correct the mistakes. The first one has been done for you.

a. A prime factor is a number that cannot be divided by a whole number greater
than one.

_____

b. A factor tree is a diagram that shows all the prime factors of a number.

_____

_____

c. The index tells us how many times to multiply the base number by itself.

_____

_____

d.HCF stands for highest calculated factor.

_____

- Fill in the missing words in these definitions of maths terms. The first one has been done for you.

a LCM stands for lowest common _____

b An integer is any positive or negative whole number or the number _____

c When the teacher asks you for a conjecture, you do not need to work out the answer,
you just need to _____

d 2^3 We say this number as 2 to the _____